

ARTICLES

Experimental study of a subsystem in an entangled two-photon state

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The state of a signal-idler photon pair of spontaneous parametric down-conversion is a typical nonlocal entangled pure state with zero entropy. The precise correlation of the subsystems is completely described by the state. However, it is an experimental choice to study only one subsystem and to ignore the other. What can we learn about the measured subsystem? Results of this kind of measurements look peculiar. The experiment confirms that the two subsystems are both in mixed states with entropy greater than zero. One can only obtain statistical knowledge of the subsystems in this kind of measurement. [S1050-2947(99)02610-4]

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One of the most surprising consequences of quantum mechanics is the entanglement of two or more distant particles. The first example of a two-particle entangled state was suggested by Einstein, Podolsky, and Rosen (EPR) in their famous *gedankenexperiment* in 1935 [1]. The EPR state is a pure state of two spatially separated particles which can be written as

$$|\Psi\rangle = \sum_{a,b} \delta(a+b-c_0)|a\rangle|b\rangle, \quad (1)$$

where a and b are the momentum or the position of particle 1 and 2, respectively, and c_0 is a constant. It is clear that state (1) is a two-particle state; however, it cannot be factored into a product of the state of particle 1 and the state of particle 2. This type of states was defined by Schrödinger as *entangled states* [2].

One, perhaps the most easily accessible, example of an entangled state is the state of a photon pair emitted in spontaneous parametric down-conversion (SPDC). SPDC is a nonlinear optical process from which a pair of signal-idler photons is generated when a pump laser beam is incident on a nonlinear optical crystal. The signal-idler two-photon state can be calculated by first order perturbation from the SPDC nonlinear interaction Hamiltonian [3],

$$|\Psi\rangle = \sum_{s,i} \delta(\omega_s + \omega_i - \omega_p) \times \delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p) a_s^\dagger(\omega(\mathbf{k}_s)) a_i^\dagger(\omega(\mathbf{k}_i)) |0\rangle, \quad (2)$$

where ω_j , \mathbf{k}_j ($j=s,i,p$) are the frequency and wave vectors of the signal (s), idler (i), and pump (p), respectively, ω_p and \mathbf{k}_p can be considered as constants, usually a single-mode laser is used for pump, and a_s^\dagger and a_i^\dagger are the respective creation operators for the signal and idler photon. The δ

functions of the state ensure energy and momentum conservation. It is indeed the conservation laws that determine the values of an observable for the pair. Quantum mechanically, state (2) only provides precise momentum (energy) *correlation* of the *pair* but no precise momentum (energy) determination for the signal photon and the idler photon. In EPR's language: the momentum (energy) of neither the signal nor the idler is determined by the state; however, if one is known to be at a certain value the other one is determined with certainty. Notice also that state (2) is a pure state. It provides a complete description of the entangled two-photon system.

Following the creation of the pair, the signal and idler may propagate to different directions and be separated by a considerably large distance. If it is a free propagation, the state will remain unchanged except for the gain of a phase, so that the precise momentum (energy) *correlation* of the *pair* still holds. The conservation laws guarantee the precise value of an observable with respect to the pair (not to the individual subsystems). It is in this sense that we say that the entangled two-photon state of SPDC is *nonlocal*. Quantum theory does allow a complete description of the precise *correlation* for the spatially separated subsystems, but no complete description for the physical reality of the subsystems defined by EPR. It is in this sense that we say that quantum mechanical description of the entangled system is *nonlocal*.

So far, our discussion involves no measurement.

In a type of measurements when "joint detections" are involved (for example, a coincidence detection for the SPDC pair), it is the intensity correlation, $\langle \Psi | \hat{I}_1 \hat{I}_2 | \Psi \rangle$, or the fourth order correlation of the fields, $\langle \Psi | \hat{E}_1^{(-)} \hat{E}_2^{(-)} \hat{E}_2^{(+)} \hat{E}_1^{(+)} | \Psi \rangle$, which is measured. One may include spin (polarization, for photon) correlations in the coincidence joint detection too, as in the measurements for the EPR-Bohm state [4]. If the correlations of the pair have been built up in the entangled two-particle state from the beginning, it comes as no surprise that the intensity correlation reflects *perfect* correlation (EPR, EPR-Bohm, or EPR-Bell type correlation) of the pair: the distance between the detectors would not matter. The entangled state indeed indicates and represents a very different

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physical reality: Does the signal photon or idler photon have a defined momentum (energy) in state (2)? No. Does the pair have a defined *total momentum (energy)* in state (2)? Yes. In state (2) the precise value of an observable is determined in the form of *total value* by conservation laws. In addition, one cannot “imagine” two individual wave packets, one associated with the signal photon and the other associated with the idler photon. It is a nonfactorizable two-dimensional “wave packet” associated with the entangled two-particle system [5]. For this reason we call the signal-idler pair the “*biphoton*.” Many interesting phenomena involving biphotons have been demonstrated in two-photon interferometry and in two-photon correlation type experiments [6]. Several recent experiments have clearly shown that the two-photon interference is not the interference between two photons. It is not the signal and idler photon wave packets but the two-dimensional biphoton wave packet that plays the role [7].

However, an experimentalist can choose to look at one part of the entangled system and to ignore the other. The subsystems may be separated spatially. For instance, one can use a photon counting detector to register a “click” event of the signal photon of the two-photon state of SPDC (not a “click-click” event.) to study only the properties of the signal and leave the idler undisturbed or to be predicted. What can we predict for the idler photon in this kind of measurement? Before answering this question, it may be better to ask first: “what can we learn about the signal photon in this kind of measurement?”

An interesting situation arises: while the two-photon state of SPDC is a pure state, the respective states of the signal and idler photon are not. The states of the individual signal and idler are both in thermal (mixed) states. This has been pointed out by several researchers, e.g., [8–12], from different perspectives. The significance of the fact that two parts each in a mixed state constitute a quantum mechanical system in a pure state was emphasized by Yurke and Potasek [9] as an example of purely quantum thermalization, that is, obtaining mixed states out of pure states in a Hamiltonian system. Cerf and Adami [11] introduced the *mutual* ($S_{A:B} = S_{B:A}$) and *conditional* ($S_{A|B}$, $S_{B|A}$) entropy (or information) for a two-particle system similar to the mutual and conditional entropies as defined in classical probability theory:

$$S = S_{A|B} + S_{B|A} + S_{A:B}, \quad S_A = S_{A|B} + S_{A:B},$$

$$S_B = S_{B|A} + S_{A:B}. \quad (3)$$

For an entangled two-particle system in a pure state (so that $S=0$), the relations in Eq. (3) give

$$S_A + S_{B|A} = 0, \quad S_B + S_{A|B} = 0. \quad (4)$$

The paradox of the whole system entropy S being zero while an entropy of either of its parts S_A or S_B is positive (which is a formal expression of the statement that the information contained in the whole system is less than the information contained in its parts) is suggested to be resolvable by allowing the conditional entropy to take on negative values.

In this paper we report experimental work along the lines of this discussion. The reported experiment hinges on a typical Fourier spectroscopy measurement. The schematic setup

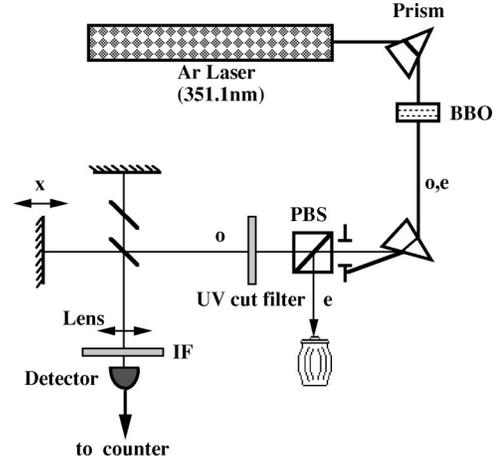


FIG. 1. Schematic of the experimental setup. A Michelson interferometer is used to study the spectrum of the signal of SPDC. A bandpass spectral filter centered at 702.2 nm with 83 nm FWHM of a Gaussian transmittance function is placed in front of the photon counting detector.

is shown in Fig. 1. The measurement is based on a “click” type single-photon detection; however, the photon source is an entangled two-photon source of SPDC: a 3 mm BBO (β -BaB₂O₄) crystal pumped by a 351.1 nm cw argon ion laser line. The orthogonally polarized signal-idler photon pairs are generated satisfying the collinear degenerate (centered at wavelength 702.2 nm) type-II phase matching condition [13]. The idler (extraordinary ray of BBO) is removed by a polarizing beam splitter (PBS). The signal (ordinary ray of BBO) is then sent to a Michelson interferometer. A photon counting detector is coupled to the output port of the interferometer through a 25 mm focal lens. A 702.2 nm spectral filter with Gaussian transmittance function [bandwidth 83 nm full width at half maximum (FWHM)] is placed in front of the detector. The counting rate of the detector is recorded as a function of the optical arm length difference ΔL of the Michelson interferometer.

The experimental data are reported in Fig. 2. The envelope of the sinusoidal modulations (in segments) is fitted very well by two “notch” functions (upper and lower part of the envelope). We find that this fit is better than a fit using Gaussian functions. The width of the triangular base is about 225 μ m, which corresponds to roughly a spectral bandwidth of 2.2 nm.

To find an explanation of this result, we must first examine the two-photon state of SPDC. The state of the signal photon is obtained by taking a partial trace of the two-photon state density operator, integrating over the spectrum of the idler, and vice versa:

$$\hat{\rho}_s = \text{tr}_i \hat{\rho}, \quad \hat{\rho}_i = \text{tr}_s \hat{\rho}, \quad (5)$$

with

$$\hat{\rho} \equiv |\Psi\rangle\langle\Psi|, \quad (6)$$

where $\hat{\rho}$ is the density matrix operator and $|\Psi\rangle$ is the two-photon state (2).

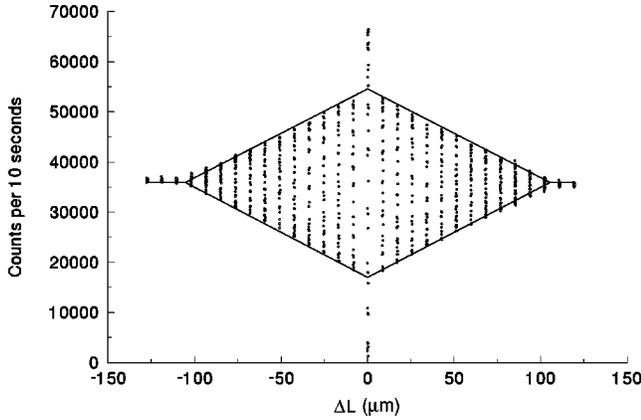


FIG. 2. Experimental data indicated a “double notch” envelope of the interference pattern. The X axis, ΔL in μm , is the optical arm difference of the Michelson interferometer. Each of the dotted single vertical segments contains many cycles of sinusoidal modulations. The spike at $\Delta L = 0$, usually called “white light condition” for observing “white light” interference, is a broadband interference pattern which is determined by the spectral filter.

First, it is very interesting to find that even though the two-photon EPR state of SPDC is a pure state, i.e., $\hat{\rho}^2 = \hat{\rho}$, the corresponding single-photon state of the signal and idler are not, i.e., $\hat{\rho}_{s,i}^2 \neq \hat{\rho}_{s,i}$. This agrees with the earlier mentioned fact that the entropy of the system is zero (pure state) while each subsystem has an entropy greater than zero (mixed state). The zero entropy condition for a system in a pure state reflects the fact that the quantum state $|\Psi\rangle$ provides a *complete* description of the system. On the other hand, the mixed state of each subsystem only reveals its statistical nature.

In the experiment, we realize a collinear degenerate type-II phase matching [13]. This means that the SPDC crystal orientation is such that the orthogonally polarized signal-idler pair with degenerate frequency $\omega = \omega_p/2$ is emitted collinear. We select this direction by a set of pinholes during the experimental alignment process. Then the integral in Eq. (2) can be simplified to an integral over a frequency detuning parameter ν (the detailed calculation can be found in Ref. [5]):

$$|\Psi\rangle = A_0 \int d\nu \Phi(DL\nu) a_s^\dagger(\omega + \nu) a_i^\dagger(\omega - \nu) |0\rangle, \quad (7)$$

where the sinc-like function $\Phi(DL\nu)$ follows from Eq. (2) considering a finite length of the SPDC crystal [3]. It represents a frequency spectrum of the two-photon state,

$$\Phi(DL\nu) = \frac{1 - e^{-iDL\nu}}{iDL\nu}, \quad (8)$$

which is determined by the finite crystal length L and, specifically for the collinear degenerate type-II SPDC, by the difference of inverse group velocities for the signal (ordinary ray) and the idler (extraordinary ray): $D \equiv 1/u_o - 1/u_e$.

The constant A_0 is found from the normalization condition $\text{tr } \rho = \langle \Psi | \Psi \rangle = 1$:

$$A_0 = \sqrt{\frac{DL}{4\pi}}.$$

Substituting $|\Psi\rangle$ in the form of Eq. (7) into Eq. (5), the density matrix of the signal is calculated to be

$$\hat{\rho}_s = A_0^2 \int d\nu |\Phi(\nu)|^2 a_s^\dagger(\omega + \nu) |0\rangle \langle 0| a_s(\omega + \nu), \quad (9)$$

where

$$|\Phi(\nu)|^2 = \text{sinc}^2 \frac{DL\nu}{2}. \quad (10)$$

In Eq. (9) we consider a multimode (a continuous frequency spectrum) entangled system with a single quantum, $n = 1$. The operator (9) describes the statistical distribution of this quantum. This is a good approximation since the coupling in SPDC is weak and greater number states $n > 1$ that correspond to higher perturbation orders are extremely unlikely. On the other hand, $n = 0$ represents vacuum fields that do not result in detections [14].

Now we can understand very well the experimental results. (i) For a spectrum of sinc^2 function we do expect a double “notch” envelope in the measurement and the base of the triangle, which is determined by DL , is calculated to be $225 \mu\text{m}$ (we have considered the optical path difference is twice the arm difference in the Michelson interferometer), corresponding to a 2.2 nm bandwidth. The experimental result, from fitting, is about 225 nm , which agrees well with the prediction [15]. (ii) We see that the spectrum of the signal photon is dependent on the group velocity of the idler photon, which is not measured at all in our experiment. However, this comes as no surprise, because the state of the signal photon is calculated from the two-photon state by integrating over the idler modes. (iii) We also see immediately that $\hat{\rho}_s^2 \neq \hat{\rho}_s$, so the signal and idler single-photon states are both mixed states. It is then straightforward to evaluate numerically the von Neuman entropy S [17] of the signal (or idler) subsystem,

$$S_s = -\text{tr}[\hat{\rho}_s \ln \hat{\rho}_s], \quad (11)$$

based on the “double notch” fitting function. Note that operator (9) is diagonal. Taking its trace is simply performing an integration over the frequency spectrum with the spectral density of Eq. (10). To compute the integral of Eq. (11) for the density matrix $\hat{\rho}_s$ of Eq. (5), we replace variable ν by a dimensionless variable $DL\nu/2$ and evaluate the integral numerically. The calculation yields

$$S_s \approx 6.4 > 0.$$

This again indicates the statistical mixed nature of the state of a photon (subsystem) in an entangled two-photon system.

Based on the experimental data, we conclude that the entropy of signal and idler are both greater than zero (mixed state), while the entropy of the signal-idler two-photon system is zero (pure state). This may mean that negative entropy is present somewhere in the system, perhaps in the form of the conditional entropy [11]. By the definition of the condi-

tional entropy, one is tempted to say that *given the result of a measurement over one particle, the result of measurement over the other must yield negative information*. This paradoxical statement is similar to and in fact closely related to the EPR “paradox.” We suggest that the paradox comes from the same philosophy.

In the kind of measurements when the experiment only involves a subsystem of an entangled multiparticle system and leaves the remaining parts undisturbed, one can only

obtain statistical knowledge of the subsystems. Neither the measured subsystem nor the remaining parts is in a pure state. The individual subsystems are described statistically by the quantum theory before the measurement and after the measurement.

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