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Experimental entanglement concentration and universal Bell-state synthesizer

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We report a Bell-state synthesizer in which an interferometric entanglement concentration scheme is used. An initially mixed polarization state from type-II spontaneous parametric down-conversion becomes entangled after the interferometric entanglement concentration. This Bell-state synthesizer is universal in the sense that the output polarization state is not affected by spectral filtering, crystal thickness, and, most importantly, the choice of pump source. It is also robust against environmental disturbance and a more general state, partially mixed–partially entangled state, can be readily generated as well.

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I. INTRODUCTION

Multiparticle quantum entanglement traditionally has been associated with fundamental issues in quantum physics, such as, the uncertainty principle and the locality, reality, and causality problems in quantum theory [1]. Recently, quantum entanglement has found its applications in metrology, communication, and information processing [2]. To successfully implement these new ideas, one must be able to generate and manipulate entangled states at will. It is, however, generally recognized that even the generation, not to mention manipulation, of multiparticle entanglement is not trivial. Nonetheless, a great deal of work has been carried out with entangled two-qubit quantum states, or Bell states. These states are important not only because of their simplicity, but also because of their utility in applications, such as, quantum cryptography. In addition, entangled two-qubit states may one day serve as building blocks for the construction of states of three or more entangled qubits [3].

The first direct generation of an entangled two-qubit state involved photon pairs produced in the process of cw-pumped type-II spontaneous parametric down-conversion (SPDC) [4,5]. Although this method is still widely used for the generation of polarization entangled states, it has its limitations. In particular, the photon pair emission times are completely random. This is a drawback in applications such as quantum teleportation, multiphoton state generation, practical quantum cryptography, etc., where knowledge of the approximate times of emission is required.

Much of the uncertainty in emission time is eliminated when the SPDC process is pumped by an ultrafast laser. Unfortunately, differences in the spectral and temporal properties of the photon pair cause the polarization entanglement to suffer with this type of pumping scheme [6,7]. It is possible to ''concentrate'' (following the definition in Ref. [8]) the entanglement by passing the photons through narrow spectral filters, effectively retaining only the more highly entangled pairs. The entanglement concentration based on a local filtering process such as this is not desirable, however, since most of the photons are simply wasted. A number of schemes involving multiple crystals have been devised to circumvent these problems [9], although none can match the simplicity and stability of a single-crystal scheme.

In this paper, we report the experimental demonstration of a general entanglement concentration scheme in a two-qubit state of type-II SPDC. Our entanglement concentration scheme does not rely on local filtering. Therefore, the degree of entanglement is not affected by the pump bandwidth, the thickness of nonlinear crystal, the bandwidth of spectral filters, etc. As a result, no photons are wasted: all qubit pairs, which are initially in a mixed state, exit the entanglement concentrator as entangled qubit pairs.

II. ENTANGLEMENT CONCENTRATION SCHEME

Consider the polarization state of the photon pair generated from a type-II BBO crystal pumped either by a cw or by an ultrafast pump laser, see Fig. 1(a). As in Ref. [4], attention is restricted to the intersections of the cones made by the *e* and *o* rays exiting the BBO crystal. In each of these two directions, a photon of either polarization (horizontal or vertical) may be found, with the orthogonal polarization found in the conjugate beam. Unlike common misconcep-

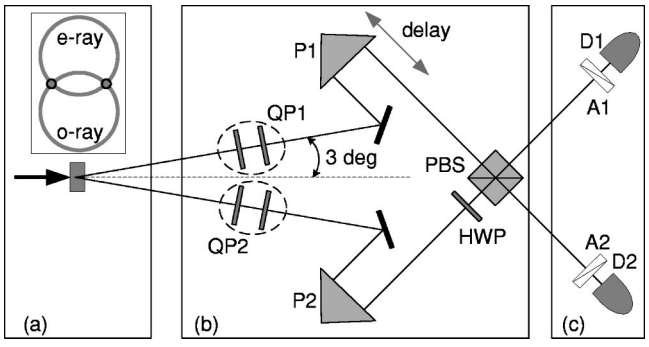

FIG. 1. (a) Noncollinear type-II SPDC is used to prepare an initial two-qubit mixed state. (b) Entanglement concentration scheme. (c) Detectors and polarization analyzers.

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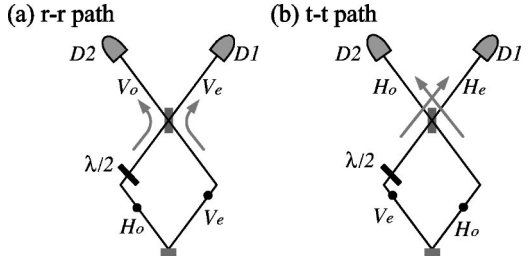

FIG. 2. Two possible quantum-mechanical paths that a photon pair may follow.

tion, the photon pairs found in these two directions are not polarization entangled. In fact, the polarization state of the photon pair is best represented as the mixed state $\rho_{mix} = \frac{1}{2}(|H_1\rangle|V_2\rangle\langle V_2|\langle H_1| + |V_1\rangle|H_2\rangle\langle H_2|\langle V_1|)$, where $|H\rangle$ and $|V\rangle$ refer to the horizontal and vertical polarization state of a single photon, respectively.

The reason the state is mixed has to do with timing information carried by the photon pair. Because the group velocities experienced by the different polarizations are not the same, one polarization always precedes the other. Thus, the two amplitudes $|H_1\rangle|V_2\rangle$ and $|V_1\rangle|H_2\rangle$ are distinguishable in principle. In cw-pumped type-II SPDC, a pair of birefringent compensators, with the effective thickness equal to half the down-conversion crystal, can remove this timing information, thus transforming the mixed state to a pure polarization entangled state [4,10].

But in ultrafast type-II SPDC, the pump pulse introduces additional timing information, which cannot be eliminated with the birefringent compensators [6]. It is therefore impossible to transform the mixed state into a pure polarization entangled state [7]. The interferometric entanglement concentration scheme shown in Fig. 1(b), nevertheless, allows us to transform the mixed state to a polarization entangled state, regardless of the pump bandwidth, crystal thickness, and spectral filters used [11].

Let us now discuss the entanglement concentration scheme in detail, see Fig. 1(b). The photon pairs exit the crystal and travel equal distances (delay $\tau=0$) to a polarization beam splitter (PBS). A $\lambda/2$ plate inserted in one arm rotates the polarization by 90° and ensures that the photon pairs have the same polarization when they reach the PBS. Thus, there are two possible outcomes: both photons are reflected (r-r); or both are transmitted (t-t). These two biphoton paths are illustrated in Fig. 2. Since it is never the case that two photons exit the same port of the PBS, no postselection is required. Note also that the photon that leaves the crystal with e polarization (o polarization) is always detected by D_1 (D_2). Therefore, the intrinsic timing information present in type-II SPDC cannot be used to distinguish between the t-t and the r-r paths. As a result, the temporal and spectral differences between the photon pair have no bearing on the polarization entanglement. We, therefore, have effectively ''disentangled'' the timing information and the polarization information of the qubit pair. (Note that the photon pair arrival times are still known within the coherence time of the pump pulse.)

The resulting amplitudes $|H_1\rangle|H_2\rangle$ and $|V_1\rangle|V_2\rangle$ are then distinguishable only by polarization. Therefore, they are in quantum superposition $|\Phi\rangle = 1/\sqrt{2}(|H_1\rangle|H_2\rangle + e^{i\varphi}|V_1\rangle|V_2\rangle)$. In this expression, φ is the phase between the two terms and may be varied by tilting the phase plates QP1 and QP2. With a setting of $\varphi=0$, the density matrix of the output state can be written as $\rho_{ent} = |\Phi^{(+)}\rangle\langle\Phi^{(+)}|$.

If the delay τ is introduced in one arm relative to the other, then the overall overlap between the two amplitudes becomes smaller. In this case, a more general state, partially mixed–partially entangled state, $\rho = \varepsilon\rho_{ent} + (1-\varepsilon)\rho_{mix}$, where $0 \leqslant \varepsilon \leqslant 1$, is generated.

III. UNIVERSAL BELL-STATE SYNTHESIZER

To understand why this entanglement concentration scheme works as a universal Bell-state synthesizer, it is necessary to carry out a quantum-mechanical calculation of the joint detection rate. What we would like to show here is that the quantum interference at $\tau=0$ (balanced interferometer) does not depend on any parameters that are related to the pump source, spectral filtering, and the crystal properties. We present a brief summary of the calculation here, with a detailed calculation to be published elsewhere [12].

The coincidence count rate R_c has the form

$$R_c \propto \int dt_1 dt_2 |\langle 0|E_1^{(+)}(t_1,\tau)E_2^{(+)}(t_2,\tau)|\psi\rangle|^2,$$

where $|\psi\rangle$ is the state of type-II SPDC [6]. Assuming that the quartz phase plates are adjusted so that $\varphi=0$, the electric-field operators that reach the detectors in this experiment can be written as

$$E_1^{(+)}(t_1,\tau) = \int d\omega' \{\cos\theta_1 e^{-i\omega' (t_1+\tau)} a_{Ve}(\omega') - \sin\theta_1 e^{-i\omega' t_1} a_{He}(\omega')\},$$

$$E_2^{(+)}(t_2,\tau) = \int d\omega' \{\cos\theta_2 e^{-i\omega' t_2} a_{Vo}(\omega') - \sin\theta_2 e^{-i\omega' (t_2+\tau)} a_{Ho}(\omega')\},$$

where, for example, $a_{Vo}(\omega')$ is the annihilation operator for a photon of frequency ω' with vertical polarization which was originally created as the o ray of the crystal. θ_1 and θ_2 are the angles of the polarization analyzers A_1 and A_2, respectively.

Upon carrying out the calculation, we find that the coincidence count rate has the form

$$R_c \propto \int dt_+ dt_- |\cos\theta_1 \cos\theta_2 \Pi(t_+ + \tau/2,\, t_- + \tau) + \sin\theta_1 \sin\theta_2 \Pi(t_+ + \tau/2,\, t_- - \tau)|^2. \quad (1)$$

The detailed expression of $\Pi(t_+, t_-)$ is given in Refs. [9,12] and it contains all the important factors such as crystal length, pump bandwidth, bandwidth of spectral filters, etc.

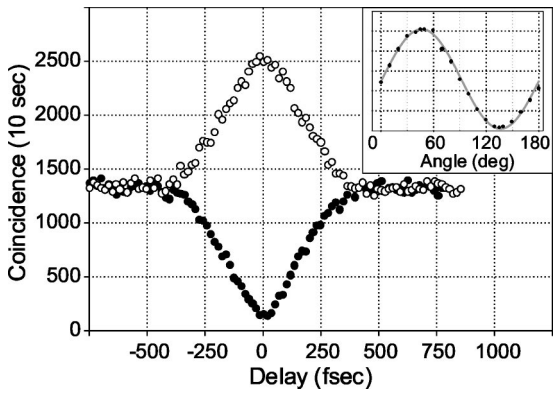

FIG. 3. Experimental data for cw-pumped type-II SPDC. $\varphi=0$ for open circle ($|\Phi^{(+)}\rangle$) and $\varphi=\pi$ for solid circle ($|\Phi^{(-)}\rangle$). Inset shows the polarization interference for $|\Phi^{(+)}\rangle$ state.

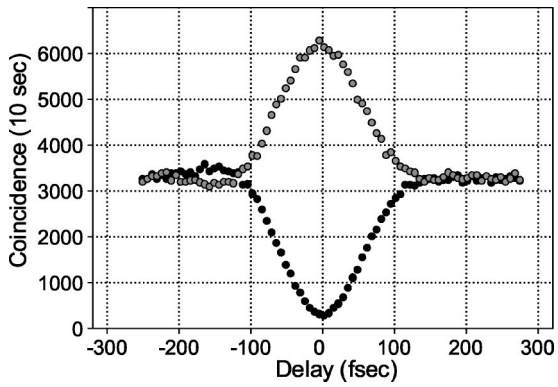

FIG. 4. Experimental data for ultrafast type-II SPDC. Analyzer angles were A_1 (A_2)$=45°$ ($45°$) for the peak and $45°$ ($-45°$) for the dip.

Let us now look at Eq. (1) carefully. If the interferometer is balanced, i.e., $\tau=0$, $\Pi(t_+,t_-)$ factors out so that $R_c \propto \cos(\theta_1-\theta_2)^2$, a clear signature of the polarization entangled state $|\Phi^{(+)}\rangle$. This means that, when the $\tau=0$, all parameters that affect the temporal shape of the $\Pi(t_+,t_-)$, including the pump bandwidth, the crystal properties, the crystal thickness, and the filter bandwidth, simply do not have any effect on the quantum interference. For this reason, this scheme can be considered as a universal Bell-state synthesizer.

IV. EXPERIMENT

As described above, the initial polarization state, a mixed state, was prepared by type-II noncollinear frequency-degenerate SPDC. A 3-mm BBO crystal was pumped by a cw argon ion laser operating at 351.1 nm, producing photons with a central wavelength of 702.2 nm. The delay τ was introduced through the motion of one of the trombone prisms and the phase between the two alternatives (shown in Fig. 2) was adjusted by slightly tilting the 600-μm quartz plates QP1 in opposite directions. The optic axes of the quartz crystals were oriented vertically. The outputs from the two detectors were fed to a time-to-amplitude converter (TAC) and the TAC output was analyzed by a multichannel analyzer with a coincidence window set to 3 nsec. No spectral filters were used in this experiment.

We first carried out typical space-time interference experiments by varying the delay τ with both polarization analyzers set at 45°. The phase term was adjusted by tilting the quartz plates QP1. The settings $\varphi=0$ and $\varphi=\pi$ were selected to prepare Bell states $|\Phi^{(+)}\rangle$ and $|\Phi^{(-)}\rangle$, respectively. Figure 3 shows the experimental data. When the delay τ is zero, i.e., no path-length difference between the two arms, complete destructive or constructive interference is observed. Note that this is different from that of typical type-II case in which τ equal to half the crystal thickness should be inserted to observe complete quantum interference [4,10]. The typical triangular two-photon wave packet is clearly demonstrated and the base width of the triangular wave packet agrees well with the theoretically expected value of 742 fsec [10,12].

The inset of Fig. 3 shows the polarization interference for $|\Phi^{(+)}\rangle$ state. A_1 was fixed at 45° and A_2 was rotated. The expected $\cos(\theta_1-\theta_2)^2$ correlation is clearly demonstrated.

The above space-time and polarization measurements were repeated with several different spectral filters with almost no change in the quality of quantum interference. The stability of the interferometer was also checked by repeating the polarization interference measurement at different times. We have found almost no change in the visibility of the polarization interference. This is due to the fact that a small change around $\tau=0$ has very little effect on the quantum interference, as shown in Fig. 3.

We have also carried out the experiment with an ultrafast laser used as the pump. A second harmonic (390 nm) of a commercial mode-locked Ti:Sa laser operating at 780 nm was used as a pump. The pump pulse duration was about 130 fsec. The thickness of the type-II BBO crystal was 3 mm and the SPDC radiation was centered at 780 nm. Interference filters with 20-nm full width at half maximum (FWHM) were inserted in front of each detector to reduce noise counts. The effective coincidence window for this experiment was 3 nsec. Figure 4 shows a typical data set for this measurement. The observed visibility is greater than 90%, by far the highest in ultrafast type-II SPDC generated from a thick crystal (for comparison, see Ref. [7] for usual ultrafast type-II SPDC data). In principle, the visibility does not depend on crystal, pump, and filter parameters, as shown in Eq. (1). The FWHM of the interference is roughly estimated to be 140–150 fsec, which is very close to the theoretically expected value of 160 fsec (assuming exactly 2 nm FWHM pump bandwidth) [12].

V. DISCUSSION

It is helpful to introduce the terms ''entanglement'' or ''entanglement of formation'' (E) and ''entropy'' or ''entropy of entanglement'' (S) formally, as done in Ref. [8], to visualize the entanglement concentration process in this work.

Under these definitions, our initial mixed two-qubit state has $S=0.5$ and $E=0$ [13]. The output state of the balanced entanglement concentrator has $E=1$ and $S=0$. Most importantly, the entanglement concentration has been obtained

without discarding any subensemble of the initial two-qubit system.

A more general state, partially mixed–partially entangled, can be readily prepared by introducing the delay τ in one arm. If $\tau \neq 0$ and $\tau < |\delta t|$, where δt is the two-photon coherence length, we obtain a state which is not completely entangled and not completely mixed. Such states are called Werner states [14] and lie between two points $(S,E) = (0.5,0)$ and $(S,E) = (0,1)$. Only recently, researchers have started to study these states experimentally in the cw domain [15]. We have recently finished tomographic measurements of several such Werner states in pulsed two-photon polarization states using this scheme and the results will be published elsewhere [16]. Such states are important in studying controllable decoherence in multiqubit systems (for now, however, it is limited to two-qubit systems). Since our scheme readily offers controllable decoherence in the pulsed domain, we believe that it will be a useful tool to generate a multiqubit entangled state and to study its decoherence in a controlled environment.

In conclusion, we have reported the experimental realization of entanglement concentration without local filtering and demonstrated a universal Bell-state synthesizer. Although typical type-II SPDC is used as a source, our Bell-state synthesizer is not affected by the pump source, the crystal properties, and the use of spectral filters. In addition, a more general state, partially mixed–partially entangled state can be readily generated. We believe that this new Bell-state synthesizer will be useful in experimental studies of quantum information science and as a building block of multiparticle entanglement.

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[13] $S = -\Sigma_{i=1}^{4} \lambda_i \log_4 \lambda_i$, where λ_i are the eigenvalues of the density matrix ρ. $S = 0.5$ because the initial state is a mixture of $|H_1\rangle|V_2\rangle$ and $|V_1\rangle|H_2\rangle$ of all four possible states.

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