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Single-photon two-qubit entangled states: Preparation and measurement

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We implement experimentally a deterministic method to prepare and measure the so-called single-photon two-qubit entangled states or single-photon Bell states, in which the polarization and the spatial modes of a single photon each represent a quantum bit. All four single-photon Bell states can be easily prepared and measured deterministically using linear optical elements alone. We also discuss how this method can be used for the recently proposed single-photon two-qubit quantum cryptography scheme.

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Entanglement usually refers to multiparticle quantum entanglement which exhibits nonlocal quantum correlations that are verified experimentally by observing multiparticle quantum interference [1]. For example, the two-photon entanglement in spontaneous parametric down-conversion (SPDC) photon pair is observed in the form of fourth-order quantum interference [2]. Recently, multiparticle entanglement and quantum interference effects have been shown to be essential for new quantum applications, such as quantum information, metrology, lithography, etc., Refs. [3–5].

A different type of ‘‘entanglement,’’ namely, ‘‘single-particle entanglement’’ or ‘‘entanglement’’ of internal degrees of freedom of a single quantum particle started to attract interest recently. Although single-particle entanglement lacks nonlocality which is at the heart of multiparticle entanglement necessary for a number of quantum applications mentioned above [6], it has been shown, nevertheless, to be useful for simulating certain quantum algorithms at the expense of exponential increase of required physical resources [7–9]. Several experiments illustrating this point have already been carried out [10,11]. It has also been shown that single-photon two-qubit states may be useful for deterministic cryptographic schemes [12].

For single-photon two-qubit states, two dichotomic variables of a single photon represent the two qubits. Usually, one is the polarization qubit in which the basis states are the orthogonal polarization states of the single photon (e.g., horizontal $|H\rangle$ or vertical $|V\rangle$ polarization states) and the other is the spatial qubit in which the basis states are two spatial modes of the single photon (e.g., the photon travels in path a or in path b) [6,7,9,12]. Clearly, a complete basis for the single-photon two-qubit state can be formed by a set of any four orthonormal states of the photon. For example, a set of $|a,V\rangle$, $|a,H\rangle$, $|b,V\rangle$, and $|b,H\rangle$ forms a complete (product) basis for the single-photon two-qubit Hilbert space. Preparation and measurement of such (product) basis states are trivial since interference is not required in both preparation and measurement stages. It is also possible to consider the product basis states that are composed of symmetric and antisymmetric superposition states of the polarization qubit and the spatial qubit [12]. We will discuss this case later in this paper.

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In the entangled basis of the single-photon two-qubit state, the single-photon Bell states

$$|\Psi^{(\pm)}\rangle \equiv \frac{1}{\sqrt{2}}(|a,H\rangle \pm |b,V\rangle),$$

$$|\Phi^{(\pm)}\rangle \equiv \frac{1}{\sqrt{2}}(|a,V\rangle \pm |b,H\rangle)$$

form a complete basis. In this paper, we propose a deterministic method to prepare and measure the ‘‘single-photon Bell states,’’ report results on the experimental implementation of the method, and discuss some potential difficulties related to the single-photon two-qubit quantum cryptography scheme proposed in Ref. [12].

The outline of the experimental setup is shown in Fig. 1. Let us first focus on the state preparation part shown in Fig.

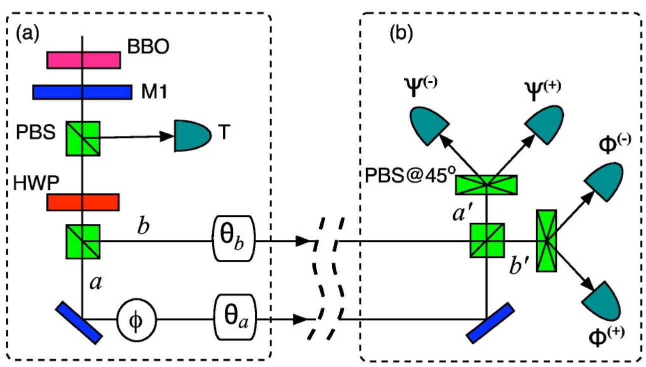

FIG. 1. Outline of the experiment. (a) Preparation of a single-photon two-qubit entangled state (Bell state) is done by using a single photon polarized in 45° and a PBS. The single-photon state is prepared by detecting one photon of the SPDC photon pair with the trigger detector T. The HWP rotates the polarization of the single photon to 45° and the second PBS prepares a single-photon Bell state. Additional phase and polarization elements, ϕ, θ_a, and θ_b, may be used to prepare the other three Bell states. (b) Bell-basis measurement. A PBS, which mixes spatial modes a and b, is followed by a 45° oriented PBS located at each output port. The detector clicks at the outputs of 45° oriented PBS's uniquely identify four single-photon Bell states. The preparation and measurement stages together form an equal-path Mach-Zehnder interferometer.

1(a). The single-photon state used in this experiment was generated using the postselection method first demonstrated in Ref. [13]. A 2-mm-thick type-II BBO crystal was pumped with a 351.1-nm argon laser. Orthogonally polarized SPDC photon pairs generated in the crystal had a central wavelength of 702.2 nm and propagated collinearly with the pump beam. After removing the pump laser beam with a dichroic mirror $M1$, the vertically polarized photon was directed to the trigger detector T by a polarizing beam splitter PBS and the trigger signal indicated that there was one and only one photon (polarized horizontally) traveling in the other output ports of PBS [14]. Note that this kind of postselected single-photon state has recently been used for demonstrating linear optical quantum logic gates and memory [16].

A half-wave plate (HWP) oriented at 22.5° rotated the polarization of the horizontally polarized single photon to 45° polarization state just before the second PBS. After the second PBS, the state of the single-photon can be written as

$$|\Psi^{(+)}\rangle=(|a,H\rangle+|b,V\rangle)/\sqrt{2},$$

which is a single-photon Bell state. The other three single-photon Bell states can be prepared by using an additional phase shifter ϕ and polarization rotating half-wave plates θ_a and θ_b. The spatial phase ϕ can be introduced, for example, by slightly moving the mirror in path a and it determines whether the two amplitudes in a single-photon Bell state interfere constructively ($\phi=0$) or destructively ($\phi=\pi$). The half-wave plates, indicated as θ_a and θ_b, inserted in the beam paths a and b may either flip the polarization ($\theta_a=\theta_b=45°$) or do nothing ($\theta_a=\theta_b=0°$). Therefore, all four single-photon Bell states can be easily prepared in this setup by suitable combinations of the spatial phases and polarization flip.

Note that it is also possible to prepare single-photon two-qubit product states using this scheme. To prepare single-photon two-qubit product states in the basic qubit bases ($|a\rangle$, $|b\rangle$, $|H\rangle$, and $|V\rangle$), we need to control the HWP orientations (0° or 45°, but not 22.5°) and the angles of θ_a and θ_b (0° or 45°). For example, 0° HWP angle and $\theta_a=0°$ prepares the state $|a,H\rangle$. The spatial phase ϕ is not relevant in this case because interference does not play any role.

We can also prepare single-photon two-qubit product states in a superposition basis, as proposed in the single-photon two-qubit quantum cryptography scheme [12],

$$(|a,+45°\rangle,|b,-45°\rangle,|S,V\rangle,|A,H\rangle),$$

where

$$\left.\begin{matrix}|S\rangle\\|A\rangle\end{matrix}\right\}=\frac{1}{\sqrt{2}}(|a\rangle\pm|b\rangle),\qquad \left.\begin{matrix}|+45°\rangle\\|-45°\rangle\end{matrix}\right\}=\frac{1}{\sqrt{2}}(|H\rangle\pm|V\rangle).$$

For example, $|S,V\rangle$ state can be prepared by setting the HWP at 22.5°, $\theta_a=45°$, and $\theta_b=0°$.

This setup therefore allows the preparation of various single-photon two-qubit states, as required for the deterministic quantum cryptographic scheme proposed in Ref. [12]. Since all the phase and polarization adjusting components in this setup can be replaced with electro-optical devices, automated random switching among different states is possible.

Let us now discuss the measurement of single-photon Bell states. A complete measurement of two-photon polarization Bell states requires both nonlinear optical effects and quantum interference [17,18]. On the other hand, a complete measurement of the single-photon Bell states requires only single-photon interference effects and linear optical elements [19]. It is because entangling or interacting two separate photons requires nonlinear optical elements, but "entangling-unentangling" single-photon two-qubit states require only linear optical elements as we have seen earlier.

The single-photon two-qubit Bell-basis measurement scheme is shown in Fig. 1(b). First, we mix the spatial qubit modes, labeled as a and b, at a polarizing beam splitter. Since the single-photon Bell-basis detector relies on the single-photon interference effect, it is necessary that the paths a and b are kept equal. The polarizing beam splitter transforms the incident amplitudes in the following way:

$$|a,H\rangle\to|a',H\rangle,\quad |b,V\rangle\to|a',V\rangle,$$

$$|b,H\rangle\to|b',H\rangle,\quad |a,V\rangle\to|b',V\rangle.$$

The single-photon Bell states are then transformed by the polarization beam splitter,

$$|\Psi^{(\pm)}\rangle\to\frac{1}{\sqrt{2}}|a'\rangle(|H\rangle\pm|V\rangle)=\begin{cases}|a'\rangle|+45°\rangle\\|a'\rangle|-45°\rangle,\end{cases}$$

$$|\Phi^{(\pm)}\rangle\to\frac{1}{\sqrt{2}}|b'\rangle(|V\rangle\pm|H\rangle)=\begin{cases}|b'\rangle|+45°\rangle\\-|b'\rangle|-45°\rangle.\end{cases}$$

Clearly, a 45° oriented polarizing beam splitter (PBS@45°) inserted at modes a' and b' can separate the above states into four distinct spatial modes. The four single-photon detectors placed at the output ports of PBS@45°, shown in Fig. 1(b), therefore produce an unambiguous signal that corresponds to the input single-photon Bell state.

We have implemented experimentally the preparation and measurement scheme for the single-photon Bell states. The argon pump laser was approximately 200 mW. The distance from the second polarizing beam splitter (which is used to prepare the single-photon Bell-state) to the third polarizing beam splitter (which is used to measure the state) was about 50 cm. θ_a and θ_b angles were set by hand and ϕ was set by moving the mirror in the beam path a with a computer controlled dc motor. In the Bell-basis measurement part of the setup, a half-wave plate and a polarizing beam splitter were used instead of rotating the PBS by 45°. A multimode fiber coupled single-photon counting detector was placed at each output mode and the coincidence between the trigger detector T and the Bell-basis detector was measured. The coincidence window in this experiment was about 3 nsec.

Figure 2 shows the output of the Bell-basis detectors for four different single-photon Bell-states as the input and it clearly indicates that the Bell-basis detectors behave as expected: e.g., for $|\Psi^{(-)}\rangle$ state input, only $|\Psi^{(-)}\rangle$ detector produces a signal. The data, however, show that there are still

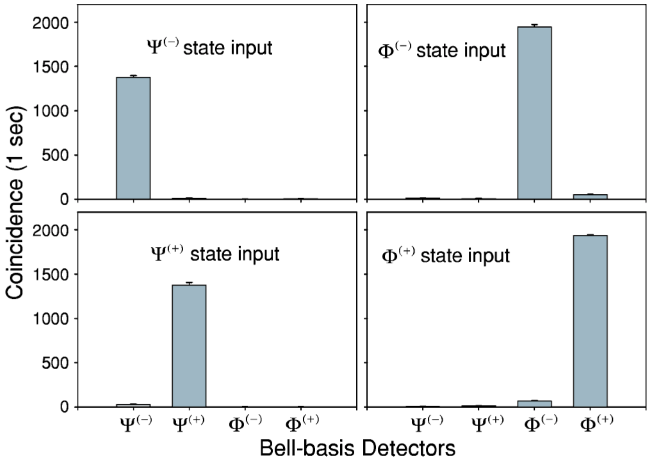

FIG. 2. Experimental data showing the outputs of Bell-state detectors for a given Bell-state input. Error counts show up due to the result of imperfect experimental alignment and phase instabilities. $|\Psi^{(\pm)}\rangle$ detectors show a lower count rate than $|\Phi^{(\pm)}\rangle$ detectors due to lower photon coupling efficiencies.

some probabilities that the single-photon ends up at a wrong Bell-basis detector. This error is the result of imperfect alignment of the experimental setup and phase instabilities of the overall Mach-Zehnder interferometer. Also, $|\Psi^{(\pm)}\rangle$ detectors show lower count rate than $|\Phi^{(\pm)}\rangle$ detectors. This is due to different photon coupling efficiencies of fiber-coupled single-photon detectors.

In this experiment, the coincidence counts between the trigger detector T and four Bell-basis detectors are measured so the dark counts of individual detectors did not show up in the data. Similar reduction of dark counts can be expected in real-world situations as well, if the single-photon source is pulsed and the Bell-basis detectors are gated accordingly.

Let us now briefly discuss how the single-photon Bell-basis detection scheme demonstrated in this paper might be applied for the single-photon two-qubit quantum cryptography scheme proposed in Ref. [12]. In their scheme, the receiver must have two detection bases; for example,

$$(|B_i\rangle)=(|a,V\rangle,|a,H\rangle,|b,V\rangle,|b,H\rangle),$$

$$(|B_i'\rangle)=(|S,+45°\rangle,|A,+45°\rangle,|S,-45°\rangle,|A,-45°\rangle).$$

The $(|B_i\rangle)$ basis detection is trivial since no interference is required for the state detection: polarizing beam splitters in paths a and b would do the job. For $(|B_i'\rangle)$ basis detection, however, interference is required. We have seen earlier that the Bell-basis detection scheme produces a unique and unambiguous output signal corresponding to the input single-photon Bell state. Therefore, for unambiguous and complete $(|B_i'\rangle)$ basis detection, we only need to find a way to transform the $(|B_i'\rangle)$ basis states to the Bell states with one-to-one correspondence in one optical setting. This may be accomplished by introducing a half-wave plate at each input port of the the polarizing beam splitter in the Bell-basis detection scheme. If the half-wave plate in path a input is oriented at 67.5° and the half-wave plate in path b is oriented at 22.5°,

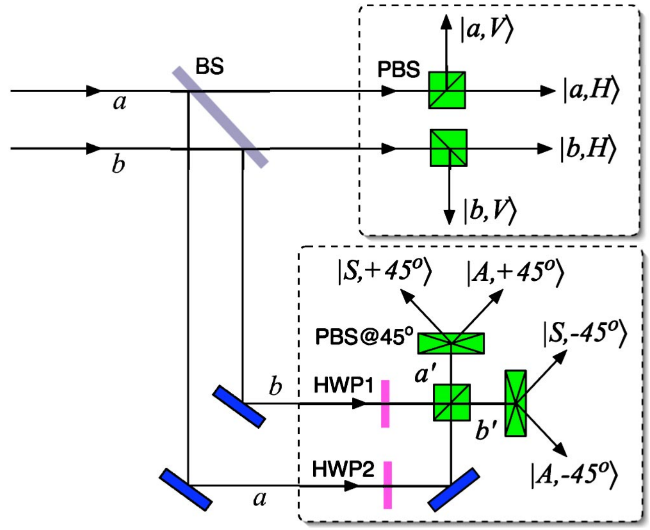

FIG. 3. Possible receiver design for the proposed single-photon two-qubit quantum cryptography scheme. Half-wave plates (HWP1 oriented at 22.5° and HWP2 oriented at 67.5°) transform the $(|B_i'\rangle)$ states into the Bell states with one to one correspondence.

the $(|B_i'\rangle)$ states are transformed to the Bell states with exact one-to-one correspondence just before the polarizing beam splitter,

$$|S,+45°\rangle\rightarrow|\Psi^{(+)}\rangle,\quad |A,+45°\rangle\rightarrow|\Psi^{(-)}\rangle,$$

$$|S,-45°\rangle\rightarrow|\Phi^{(+)}\rangle,\quad |A,-45°\rangle\rightarrow|\Phi^{(-)}\rangle.$$

Complete and unambiguous $(|B_i'\rangle)$ basis detection is therefore possible with a very simple modification of the Bell-basis detection scheme. A possible receiver design for proposed single-photon two-qubit quantum cryptography can be seen in Fig. 3.

Finally, we discuss some potential difficulties regarding quantum cryptography schemes using the single-photon two-qubit state. As we have seen in this paper, the sender and the receiver in quantum cryptography schemes using the single-photon two-qubit state are two subdivisions of a huge Mach-Zehnder interferometer. Since single-photon interference is critical for reliable and unambiguous Bell-state detections, the two spatial qubit modes a and b should be free of any phase fluctuations. If, for example, a π relative phase is temporarily introduced between the two spatial modes when the photons are traveling between the two parties, the Bell-state detectors will produce an incorrect output: for example, $|\Psi^{(-)}\rangle$ state sent to the receiver will trigger $|\Psi^{(+)}\rangle$ detector instead. Phase stability, in addition to polarization stability, therefore will be a serious issue for implementing long-distance quantum communication using the single-photon two-qubit state.

In summary, we designed and implemented a deterministic method to prepare and measure all four ‘‘single-photon two-qubit entangled states’’ or ‘‘single-photon Bell states.’’

Although the quantum cryptography scheme based on single-photon two-qubit states promises to give a factor of 2 gain compared to other cryptography scheme [12], such as the one based only on single-photon polarization states [20], the price to pay is substantially higher maintenance cost to ensure that the spatial modes are free of any phase instabilities, in addition to the polarization stability.

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[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935); J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, New York, 1987).

[2] Y.H. Shih and C.O. Alley, in *Proceedings of the Second International Symposium on Foundations of Quantum Mechanics in the Light of New Technology, Tokyo, 1986*, edited by M. Namiki *et al.* (Physical Society of Japan, Tokyo, 1987); Phys. Rev. Lett. **61**, 2921 (1988); Z.Y. Ou and L. Mandel, *ibid.* **61**, 50 (1988).

[3] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[4] D.N. Klyshko, Sov. J. Quantum Electron. **7**, 591 (1977); A. Migdall *et al.*, Appl. Opt. **37**, 3455 (1998); J. Dowling, Phys. Rev. A **57**, 4736 (1998); V. Giovannetti, S. Lloyd, and L. Maccone, *ibid.* **65**, 022309 (2002).

[5] A.N. Boto *et al.*, Phys. Rev. Lett. **85**, 2733 (2000); M. D'Angelo, M.V. Chekhova, and Y. Shih, *ibid.* **87**, 013602 (2001).

[6] R.J.C. Spreeuw, Found. Phys. **28**, 361 (1998); Phys. Rev. A **63**, 062302 (2001).

[7] N.J. Cerf, C. Adami, and P.G. Kwiat, Phys. Rev. A **57**, R1477 (1998); G. Brassard, S.L. Braunstein, and R. Cleve, Physica D **120**, 43 (1998).

[8] S. Lloyd, Phys. Rev. A **61**, 010301(R) (1999); P. Knight, Science **287**, 441 (2000).

[9] B.-G. Englert, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. A **63**, 032303 (2001).

[10] P.G. Kwiat *et al.*, J. Mod. Opt. **47**, 257 (2000); C. Dorrer *et al.*, in QELS 2001 Proceedings, QWB3 (unpublished); N. Bhattacharya *et al.*, Phys. Rev. Lett. **88**, 137901 (2002).

[11] J. Ahn, T.C. Weinacht, and P.H. Bucksbaum, Science **287**, 463 (2000); D.A. Meyer, *ibid.* **289**, 1431a (2000); P.G. Kwiat and R.J. Hughes, *ibid.* **289**, 1431a (2000).

[12] A. Beige, B.-G. Englert, C. Kurtsiefer, and H. Weinfurter, J. Phys. A **35**, L407 (2002).

[13] C.K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).

[14] One can confirm postselected single-photon states in SPDC experimentally by directly measuring the postselected photon statistics [13] or by measuring the anticorrelation parameter [15].

[15] P. Grangier, G. Roger, and A. Aspect, Europhys. Lett. **1**, 173 (1986).

[16] T.B. Pittman, B.C. Jacobs, and J.D. Franson, Phys. Rev. A **66**, 042303 (2002); T.B. Pittman and J.D. Franson, *ibid.* **66**, 062302 (2002).

[17] L. Vaidman and N. Yoran, Phys. Rev. A **59**, 116 (1999); N. Lutkenhaus, J. Calsamiglia, and K.-A. Suominen, *ibid.* **59**, 3295 (1999).

[18] Y.-H. Kim, S.P. Kulik, and Y. Shih, Phys. Rev. Lett. **86**, 1370 (2001); J. Mod. Opt. **49**, 221 (2002).

[19] This effect has been used in a recent demonstration of quantum teleportation by D. Boschi *et al.*, Phys. Rev. Lett. **80**, 1121 (1998). The qubit to be teleported, however, cannot come from outside in this case.

[20] C.H. Bennett and G. Brassard, in *Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing* (IEEE, New York, 1984); C.H. Bennett, Phys. Rev. Lett. **68**, 3121 (1992).