

Generating entangled states of two ququarts using linear optical elements

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We propose linear optical schemes for generating entangled states of two ququarts (four-dimensional quantum systems) in which the single-ququart state is constructed with frequency-nondegenerate biphoton polarization states of spontaneous parametric down-conversion. We show explicitly that it is possible to generate various two-ququart entangled states by interfering two ququarts at a linear optical beam splitter (an ordinary 50-50 beam splitter, a polarizing beam splitter, or a dichroic beam splitter).

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Two-dimensional quantum systems and entanglement (qubits and entangled qubits) have played an important role not only for the study of foundations of quantum theory but also in quantum-information research, including quantum computation, quantum teleportation, quantum dense coding, etc. D -dimensional quantum systems ($D > 2$), or qudits, have attracted much attention recently due to the potential applications in quantum-information processing and fundamental tests of quantum-theory [1–4].

It is well known that qubit states can be implemented by using any possible degrees of freedom of photons, such as polarization, path, and/or time of arrival of photons [5–7]. Just as the case of qubits, qudit states may also be implemented by using any possible degrees of freedom of photons and, recently, entanglement of two photonic qudits has been demonstrated by utilizing the high dimensionality provided by angular momentum states, transverse momentum and position, time of arrival, etc. [8–12]. It is worth noting that entangled states of two qudits in these experiments are generated directly by using angular momentum conservation in the two-photon spontaneous parametric down-conversion (SPDC) process itself [8,9] or the two-qudit entanglement is the result of transverse and/or longitudinal quantum correlations of the SPDC photon pair [10–12].

Many quantum-information applications, however, require scalable qudits (e.g., scalability to multiqubit entanglement) and, therefore, it is preferable if preparation of individual qudits and entangling operations can be separately implemented. This two-step approach is very difficult (in some cases, impossible) to implement for the qudits reported in Refs. [8–12] due to the physical nature of such photonic degrees of freedom.

In this paper, we propose linear optical schemes for generating entangled states of two photonic ququarts (four-dimensional quantum systems—i.e., $D=4$). The individual ququart in our scheme is based on frequency-nondegenerate biphoton polarization states of SPDC [13,14]. Encoding a qudit basis state on a photon pair in this way, rather than a single photon, has several unique advantages. First, the qudit basis-state measurement requires a twofold coincidence detection which makes the detection scheme quite robust

against optical noises and the effects of detector dark counts can be effectively minimized. Second, the photon pair, which is the physical carrier of qudit states, can be readily generated by SPDC. Therefore, unlike schemes that require single-photon states, there is no need to physically simulate the required photon source [15,16]. Third, since the degree of polarization of frequency-nondegenerate biphoton polarization states of SPDC is not invariant under $SU(2)$ transformations [17,18], linear optical elements (phase plates) are sufficient to switch between all four ququart bases states [13,14]. Finally, it is possible to generate entangled ququarts by using linear optical elements (beam splitters), as we shall describe below. In our scheme to generate two entangled ququarts, three types of linear optical elements will be considered: an ordinary 50-50 beam splitter (BS), a polarizing beam splitter (PBS), and a dichroic beam splitter (DBS).

Let us first briefly introduce the single-ququart-state preparation using the collinear frequency-nondegenerate biphoton polarization states of SPDC [13,14]. Since the SPDC photons have two polarization degrees of freedom, each associated with one of the two possible wavelengths determined by the phase matching condition of SPDC, it is possible to define four orthonormal two-photon states which form the bases states for a single ququart [13,14]:

$$\{|H_{\lambda_1}, H_{\lambda_2}\rangle, |H_{\lambda_1}, V_{\lambda_2}\rangle, |V_{\lambda_1}, H_{\lambda_2}\rangle, |V_{\lambda_1}, V_{\lambda_2}\rangle\}, \quad (1)$$

where the subscripts λ_1 and λ_2 refer to the two different wavelengths of the photons. Note that the photon pairs belong to the same spatial mode. Since Eq. (1) forms the four-dimensional Hilbert space for a single ququart, we may simply define the above set of bases states as

$$\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}, \quad (2)$$

such that, for example, $|H_{\lambda_1}, H_{\lambda_2}\rangle \equiv |0\rangle$.

The most general state of a single ququart can then be written as

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + c_3|3\rangle, \quad (3)$$

where $c_j = |c_j|e^{i\varphi_j}$ are the complex probability amplitudes and satisfy the normalization condition $\sum_{j=0}^3 |c_j|^2 = 1$.

The state in Eq. (3) can be realized experimentally, for example, by coherently combining four biphoton amplitudes, each representing the ququart basis state in Eq. (1). The individual biphoton amplitudes can be generated by collinear

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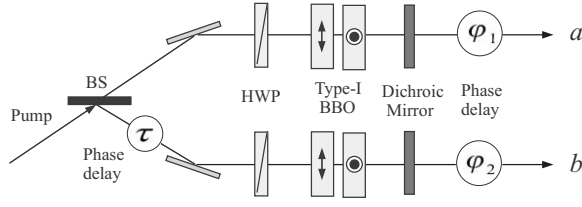


FIG. 1. One possible way to produce two biphoton ququarts in specific states. HWP is the half-wave plate, and the dichroic mirror removes the residual pump laser from SPDC photons. φ_1 and φ_2 are used to adjust the states of ququart a and ququart b , respectively. τ adjusts the relative phase delay between the two ququarts. To prepare the most general biphoton ququart state shown in Eq. (3), one would normally need two type-I and two type-II crystals in each mode. For further experimental details, see Refs. [13,14].

nondegenerate (two-color) type-I and type-II SPDC. Two type-I SPDC sources are required for generating $|H_{\lambda_1}, H_{\lambda_2}\rangle$ and $|V_{\lambda_1}, V_{\lambda_2}\rangle$. The other two amplitudes $|H_{\lambda_1}, V_{\lambda_2}\rangle$ and $|V_{\lambda_1}, H_{\lambda_2}\rangle$ can be generated with type-II SPDC. This scheme would allow full control of amplitudes and phases in Eq. (3). In real life, however, it is quite sufficient to slightly modify the collinear two-crystal setup used to prepare nondegenerate two-color Bell states in Ref. [19] for generation of an arbitrary single-ququart state. Basically, the use of collinear-nondegenerate two-color biphoton states allows us to use frequency-selective multiorder wave plates to switch among the bases states and to generate superposition states. For further details, see Refs. [13,14].

Let us now discuss linear optical schemes for entangling two biphoton ququarts. The two ququarts needed for such an experiment can be generated, for example, by using the scheme shown in Fig. 1 so that

$$|\psi\rangle_a = c_0|0\rangle_a + c_1|1\rangle_a + c_2|2\rangle_a + c_3|3\rangle_a, \quad (4)$$

$$|\psi\rangle_b = c'_0|0\rangle_b + c'_1|1\rangle_b + c'_2|2\rangle_b + c'_3|3\rangle_b. \quad (5)$$

Since we make use of quantum interference of photons at a beam splitter to generate entangled states of two ququarts, the SPDC processes must be pumped by a train of ultrashort pulses [19].

For two-ququart entangling operations using linear optical elements, consider the mode structure shown in Fig. 2. Spatial modes a and b of the beam splitter are occupied by ququart a and ququart b , respectively, so the input state is, quite obviously, a product state of two ququarts,

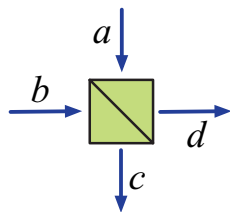


FIG. 2. (Color online) Ququart a and ququart b interfere at the beam splitter (BS, PBS, and DBS), and the joint states of ququart c and ququart d are investigated.

$$|\psi\rangle_{in} = |\psi\rangle_a \otimes |\psi\rangle_b. \quad (6)$$

Assuming the beam splitter is lossless, incident biphoton ququarts evolve unitarily to the output modes c and d . The exact form of the unitary transformation is determined by the choice of beam splitter.

In this paper, we investigate the photon states $|\psi\rangle_{out}$ at modes c and d to see if the output state $|\psi\rangle_{out}$ can be understood as two entangled ququarts, each occupying modes c and d . Three choices of beam splitter are considered: an ordinary 50-50 BS, a PBS, and a DBS.

Case I (50-50 beam splitter). The well-known unitary transformation due to a lossless 50-50 beam splitter, shown in Fig. 2, can be expressed as

$$\hat{c}_{jk} = \hat{a}_{jk} + i\hat{b}_{jk}, \quad \hat{d}_{jk} = i\hat{a}_{jk} + \hat{b}_{jk},$$

where $\hat{c}$, for example, is the annihilation operator for a photon in mode c . The subscript j and k refer to the polarization ($|H\rangle$ or $|V\rangle$) and the wavelength (λ_1 or λ_2) of the photon, respectively.

Since the single photon with a certain wavelength k has two spatial (mode a or mode b) and two polarization ($|H\rangle$ or $|V\rangle$) degrees of freedom at the input port of beam splitter shown in Fig. 2, four possible single-photon states can be expressed as

$$\hat{a}_{Hk}^\dagger|0\rangle = (1, 0, 0, 0)^T, \quad \hat{a}_{Vk}^\dagger|0\rangle = (0, 1, 0, 0)^T,$$

$$\hat{b}_{Hk}^\dagger|0\rangle = (0, 0, 1, 0)^T, \quad \hat{b}_{Vk}^\dagger|0\rangle = (0, 0, 0, 1)^T.$$

Using the above basis set, the unitary transformation for the single photon with fixed wavelength λ_1 due to the BS shown in Fig. 2 is calculated to be

$$U_{\lambda_1}^{BS} = \begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \\ i & 0 & 1 & 0 \\ 0 & i & 0 & 1 \end{pmatrix}.$$

Since the BS unitary transformation does not depend on wavelengths and polarization, $U_{\lambda_1} = U_{\lambda_2}$.

Note now that the ququart in our scheme is, in fact, a biphoton system. Two ququarts interfering at the BS, therefore, can be seen as four independent photons entering the BS via different input ports. The relevant unitary transformation in this case is calculated to be

$$U^{BS} = U_{\lambda_1}^{BS} \otimes U_{\lambda_2}^{BS} \otimes U_{\lambda_1}^{BS} \otimes U_{\lambda_2}^{BS}. \quad (7)$$

This 256×256 matrix describes all possible unitary transformations by an ordinary 50-50 beam splitter for general four-photon states entering the beam splitter in Fig. 2.

Consider two ququarts, defined in Eqs. (4) and (5), entering the input modes a and b of the BS shown in Fig. 2. It is straightforward (but lengthy) to calculate the photon state $|\psi\rangle_{out}$ is the result of rather complicated quantum interference and is calculated to be

$$|\psi\rangle_{out} = |\mathcal{O}\rangle + (c_0c'_3 + c_3c'_0 - c_1c'_2 - c_2c'_1)(|0\rangle_c|3\rangle_d + |3\rangle_c|0\rangle_d - |1\rangle_c|2\rangle_d - |2\rangle_c|1\rangle_d), \quad (8)$$

where $|\mathcal{O}\rangle$ includes all four-photon amplitudes which cannot be expressed in the ququart bases states defined in Eqs. (1) and (2). Note that there are 16 amplitudes in which the states can be expressed in the biphoton ququart basis states.

We now apply the single-ququart detection scheme discussed in Refs. [13,14] for both output modes c and d . Since the two-ququart detection scheme should involve measuring coincidence between two single-ququart detection schemes, the amplitude $|\mathcal{O}\rangle$ in Eq. (8) does not contribute to the two-ququart detection signal and hence can be ignored. The resulting postselected two-ququart state is entangled if $c_0c'_3 + c_3c'_0 - c_1c'_2 - c_2c'_1 \neq 0$ [20].

By using a bit simpler input ququart states, it is possible to generate two-ququart entangled states that are more robust and easier to manage experimentally. For example, consider two input ququarts $|\psi\rangle_a = c_0|0\rangle_a + c_3|3\rangle_a$ and $|\psi\rangle_b = c'_0|0\rangle_b + c'_3|3\rangle_b$. The output (post-selected) two-ququart entangled state is then calculated to be $|\psi\rangle_{out} = (c_0c'_3 + c_3c'_0)(|0\rangle_c|3\rangle_d + |3\rangle_c|0\rangle_d - |1\rangle_c|2\rangle_d - |2\rangle_c|1\rangle_d)$. As another example, consider two input ququarts $|\psi\rangle_a = |0\rangle_a$ and $|\psi\rangle_b = c'_0|0\rangle_b + c'_3|3\rangle_b$. The output post-selected two-ququart entangled state is calculated to be $|\psi\rangle_{out} = c'_3(|0\rangle_c|3\rangle_d + |3\rangle_c|0\rangle_d - |1\rangle_c|2\rangle_d - |2\rangle_c|1\rangle_d)$. For a BS, this turns out to be one of the simplest examples (i.e., requiring the simplest input ququart states) in which two-ququart entanglement is generated. Note that the output two-ququart entangled states are always post-selected in the linear optical entangling scheme using an ordinary 50-50 beam splitter.

Case II (polarizing beam splitter). Let us now consider using PBS instead of BS in Fig. 2. The PBS transformation can be easily described as

$$\hat{a}_{Hk} \rightarrow \hat{c}_{Hk}, \quad \hat{a}_{Vk} \rightarrow \hat{d}_{Vk},$$

$$\hat{b}_{Hk} \rightarrow \hat{d}_{Hk}, \quad \hat{b}_{Vk} \rightarrow \hat{c}_{Vk},$$

where the subscript k refers to the wavelength (λ_1 and λ_2) of the photon. Using this relation, it is found that the unitary transformation by a PBS for a single photon with wavelength λ_1 is given by a 4×4 matrix,

$$U_{\lambda_1}^{PBS} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

As before, the unitary transformation by a PBS does not depend on the wavelength of the photon, so that $U_{\lambda_1}^{PBS} = U_{\lambda_2}^{PBS}$. The unitary transformation by a PBS for four photons is then given as

$$U^{PBS} = U_{\lambda_1}^{PBS} \otimes U_{\lambda_2}^{PBS} \otimes U_{\lambda_1}^{PBS} \otimes U_{\lambda_2}^{PBS}, \quad (9)$$

which is a 256×256 matrix, describing all possible transformations by a PBS for four input photons.

If the input photon states to the PBS are two biphoton ququarts as in Eqs. (4) and (5), the output photon state in modes c and d is given as a superposition of 16 different amplitudes. Some of these amplitudes cannot be expressed in terms of biphoton ququarts in modes c and d . These amplitudes, however, do not get registered by the two-ququart detection scheme discussed earlier [20]. The post-selected two-ququart state in the output modes c and d , therefore, is given as

$$|\psi\rangle_{out} = c_0c'_0|0\rangle_c|0\rangle_d + c_1c'_1|1\rangle_c|1\rangle_d + c_2c'_2|2\rangle_c|2\rangle_d + c_3c'_3|3\rangle_c|3\rangle_d, \quad (10)$$

which is an entangled state of ququart c and ququart d .

Rather than considering the most general ququart states as the input, we may consider somewhat simpler input ququart states and this leads to drastically simpler and robust two-ququart entangled states. First, consider, two ququarts $|\psi\rangle_a = c_0|0\rangle_a + c_3|3\rangle_a$ and $|\psi\rangle_b = c'_0|0\rangle_b + c'_3|3\rangle_b$, entering the PBS. The output post-selected ququart state in this case is calculated to be $|\psi\rangle_{out} = c_0c'_0|0\rangle_c|0\rangle_d + c_3c'_3|3\rangle_c|3\rangle_d$, which is an entangled two-ququart state. This two-ququart entangled state is considerably simpler than that of BS considered earlier, even though the input states are the same.

Note that, using a BS, we have found that it is possible to obtain a (post-selected) two-ququart entangled state even though one of the input ququart is not a superposition state. For a PBS, on the other hand, this is no longer true. To obtain a two-ququart entangled state by using a PBS, both input ququart states needs to be in superposition of the same bases states.

Case III (dichroic beam splitter). Here we consider two input ququarts interfering at a DBS; see Fig. 2. A DBS is designed to reflect a photon of with wavelength λ_1 and to transmit a photon with wavelength λ_2 . Then, each photon entering a DBS in Fig. 2 is transformed in the following way:

$$\hat{a}_{j\lambda_1} \rightarrow \hat{d}_{j\lambda_1}, \quad \hat{a}_{j\lambda_2} \rightarrow \hat{c}_{j\lambda_2},$$

$$\hat{b}_{j\lambda_1} \rightarrow \hat{c}_{j\lambda_1}, \quad \hat{b}_{j\lambda_2} \rightarrow \hat{d}_{j\lambda_2},$$

where the subscript j refer to the polarization state of a photon.

For a DBS in Fig. 2, the unitary transformation on a single photon depends on the wavelength. Photons with different wavelengths, therefore, have different unitary transformations,

$$U_{\lambda_1}^{DBS} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad U_{\lambda_2}^{DBS} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and, as before, the most general unitary transformation for four photons is given by

$$U^{DBS} = U_{\lambda_1}^{DBS} \otimes U_{\lambda_2}^{DBS} \otimes U_{\lambda_1}^{DBS} \otimes U_{\lambda_2}^{DBS}. \quad (11)$$

For two ququarts, Eqs. (4) and (5), entering the DBS, the output photon state in modes c and d is calculated to be

$$\begin{aligned} |\psi\rangle_{out} = & c_0 c'_0 |0\rangle_c |0\rangle_d + c_0 c'_1 |0\rangle_c |1\rangle_d + c_2 c'_0 |0\rangle_c |2\rangle_d + c_2 c'_1 |0\rangle_c |3\rangle_d \\ & + c_1 c'_0 |1\rangle_c |0\rangle_d + c_1 c'_1 |1\rangle_c |1\rangle_d + c_3 c'_0 |1\rangle_c |2\rangle_d \\ & + c_3 c'_1 |1\rangle_c |3\rangle_d + c_0 c'_2 |2\rangle_c |0\rangle_d + c_0 c'_3 |2\rangle_c |1\rangle_d \\ & + c_2 c'_2 |2\rangle_c |2\rangle_d + c_2 c'_3 |2\rangle_c |3\rangle_d + c_1 c'_2 |3\rangle_c |0\rangle_d \\ & + c_1 c'_3 |3\rangle_c |1\rangle_d + c_3 c'_2 |3\rangle_c |2\rangle_d + c_3 c'_3 |3\rangle_c |3\rangle_d. \end{aligned} \quad (12)$$

It is interesting to note that, unlike the cases of BS in Eq. (8) and PBS in Eq. (10) considered earlier, the 16 terms in Eq. (12) are all of the nonvanishing amplitudes. No post-selection of amplitudes is necessary. On the other hand, if the input ququart states are formed by equal superposition of all ququart bases states, $c_0 = c_1 = c_2 = c_3$ and $c'_0 = c'_1 = c'_2 = c'_3$, the output state turns out to be a two-ququart product state—i.e., $|\psi\rangle_{out} = |\psi\rangle_c \otimes |\psi\rangle_d$.

It is, however, possible to generate a post-selection-free two-ququart entangled state using the DBS if the input ququart states are slight modified—for example, $|\psi\rangle_a = c_0 |0\rangle_a + c_3 |3\rangle_a$ and $|\psi\rangle_b = c'_0 |0\rangle_b + c'_3 |3\rangle_b$. The DBS output state is then calculated to be a post-selection-free two-ququart entangled state $|\psi\rangle_{out} = c_0 c'_0 |0\rangle_c |0\rangle_d + c_3 c'_0 |1\rangle_c |2\rangle_d + c_0 c'_3 |2\rangle_c |1\rangle_d + c_3 c'_3 |3\rangle_c |3\rangle_d$. Even simpler input ququarts states, such as $|\psi\rangle_a = |0\rangle_a$, $|\psi\rangle_b = c'_0 |0\rangle_b + c'_3 |3\rangle_b$ and $|\psi\rangle_a = |1\rangle_a$, $|\psi\rangle_b = c'_0 |0\rangle_b + c'_3 |3\rangle_b$, also result in experimentally manageable post-

selection-free two-ququart entangled states $|\psi\rangle_{out} = c'_0 |0\rangle_c |0\rangle_d + c'_3 |2\rangle_c |1\rangle_d$ and $|\psi\rangle_{out} = c'_0 |1\rangle_c |0\rangle_d + c'_3 |3\rangle_c |1\rangle_d$, respectively.

In summary, we have investigated linear optical schemes to generate entanglement of two ququarts, which are constructed with the biphoton polarization states of frequency-nondegenerate SPDC. We have shown that it is possible to generate various entangled states of two ququarts by interfering two biphoton ququarts at a beam splitter, such as an ordinary 50-50 beam splitter, a polarizing beam splitter, and a dichroic beam splitter [21]. The beam-splitter-based ququart entanglement scheme discussed here is in principle scalable and, therefore, it should be possible to arrange interference schemes involving multiple ququarts to generate multi-ququart entangled states.

As mentioned earlier, an experimental implementation of the proposed scheme will involve measuring fourfold coincidences [20]. Fourfold coincidence rates reported in the literature vary significantly from a few counts per hour to a few counts per second [22]. We estimate that fourfold coincidence rates of a few counts per minute (in the dichroic beam splitter scheme) should be achievable without too much difficulty by optimizing the single-ququart detection scheme discussed in Refs. [14,19].

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