PHYSICAL REVIEW A **90**, 052328 (2014)

Observation of decoherence-induced exchange symmetry breaking in an entangled state

Hyang-Tag Lim,* Jong-Chan Lee, Kang-Hee Hong, and Yoon-Ho Kim†

Department of Physics, Pohang University of Science and Technology, Pohang, 790-784, Korea

(Received 31 March 2014; published 24 November 2014)

One of the most intriguing features of entanglement is that entangled quantum systems exhibit exchange symmetry; that is, local quantum operations on the subsystems may be interchanged without affecting the quantum state. In this work, we investigate whether the exchange symmetry is preserved for the weak (or partial collapse) measurement, a type of quantum operation, in the presence of decoherence. Demonstrated using two entangled photonic polarization qubits, the experimental results clearly show that the exchange symmetry is broken once decoherence is introduced, even though the photons still share nonzero entanglement. Our results shed light on quantum state manipulation using general quantum operations on entangled quantum states in the presence of decoherence.

DOI: 10.1103/PhysRevA.90.052328   PACS number(s): 03.67.Bg, 03.65.Ud, 42.50.Ex

I. INTRODUCTION

A physical system is said to have symmetry if it is invariant with respect to a certain transformation, e.g., geometric, time reversal, particle exchange, etc. The physical symmetry not only simplifies the description of the physical system tremendously but also is regarded as one of the fundamental concepts in physics [1]. Various symmetries have been identified in different physical systems [2–5]. On the other hand, symmetry breaking is known to occur, and the broken symmetry has been associated with the emergence of novel properties of a physical system [6–11]. An entangled quantum system has been known to have an exchange symmetry; that is, local quantum operations on the subsystems may be interchanged without affecting the quantum state, which may be nonlocal [12–15].

In quantum physics, measurement has traditionally been associated with von Neumann measurement or projection measurement, in which the quantum state of the measured quantum system collapses randomly to one of the eigenstates of the measurement operator. This property arises because the measurement operators are assumed to be orthogonal to each other. In general, however, this does not need to be the case, and a more general measurement or positive operator-valued measure (POVM) may be introduced [16]. Weak (or partial collapse) measurement is an example of such a general measurement which does not fully collapse the initial quantum state to one of the eigenstates of the measurement operator [17]. Interestingly, a weak measurement may be reversed probabilistically by another weak measurement (namely, a reversing measurement), and as a result, the initial quantum state may be recovered, leading to the possibility of probabilistic quantum error correction [18]. The reversal of weak measurement for the quantum state recovery has been demonstrated using the superconducting qubit [19] and the photonic qubit [20].

Weak measurement manifests an intriguing exchange symmetry in an entangled state [12]. Consider a pair of qubits in an entangled state $|\Phi\rangle$ (see Fig. 1). As discussed above, a weak measurement performed on one subsystem can be reversed by another weak measurement (i.e., reversing measurement) on the same subsystem. Thus, the initial two-qubit entangled state $|\Phi\rangle$ is trivially recovered in the local reversal scenario shown in Fig. 1(a). Interestingly, we may apply the reversing measurement to the other qubit which had not been subjected to the weak measurement and still recover the initial entangled state $|\Phi\rangle$ [see Fig. 1(b)]. The fact that the quantum state remains the same regardless of the locations of the weak and reversing measurements clearly indicates the exchange symmetry of weak measurement in an entangled state, and this has recently been demonstrated using photonic qubits [21].

In this paper, we investigate if the exchange symmetry is preserved for weak measurement in the presence of decoherence [22]. This is a particularly interesting problem as it is known that weak measurement can be used for decoherence suppression. Decoherence suppression for a single qubit is possible by applying weak measurement and reversing measurement, respectively, before and after the decoherence channel [23,24]. Moreover, this decoherence suppression scheme can be extended to two-qubit systems [25], and recent experiments have demonstrated that weak and reversing measurements may be used to protect entanglement from decoherence [26,27], even from strong decoherence which causes entanglement sudden death [28–30]. Here, we show that the exchange symmetry of weak measurement is broken in the presence of decoherence even though the system still has nonzero entanglement.

II. THEORY

We begin by briefly introducing the concept of weak quantum measurement, its reversing properties, and the exchange symmetry. Consider a single-qubit state $|\psi_0\rangle = \alpha|0\rangle + \beta|1\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$. The weak measurement operator $W(p) = |0\rangle\langle 0| + \sqrt{1-p}\,|1\rangle\langle 1|$, where p is the strength of the weak measurement varying from 0 to 1, partially collapses the state towards $|0\rangle$, and the state after the weak measurement becomes $|\psi_1\rangle = W(p)|\psi_0\rangle = \alpha|0\rangle + \beta\sqrt{1-p}\,|1\rangle$ [20]. If we now apply another weak measurement, $R(p_r) = \sqrt{1-p_r}\,|0\rangle\langle 0| + |1\rangle\langle 1|$, which partially

*forestht@gmail.com
†yoonho72@gmail.com

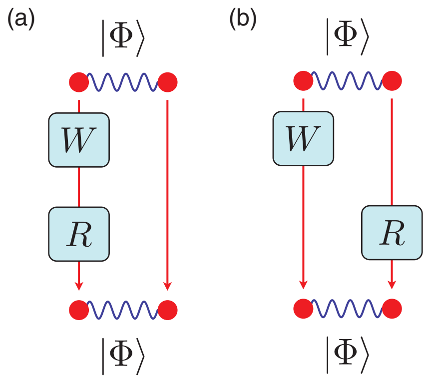

FIG. 1. (Color online) (a) Local reversal: weak and reversal measurements are applied to the same qubit. (b) Nonlocal reversal: weak and reversal measurements are applied to different qubits. Both give the same results, fully recovering the initial quantum state $|\Phi\rangle$.

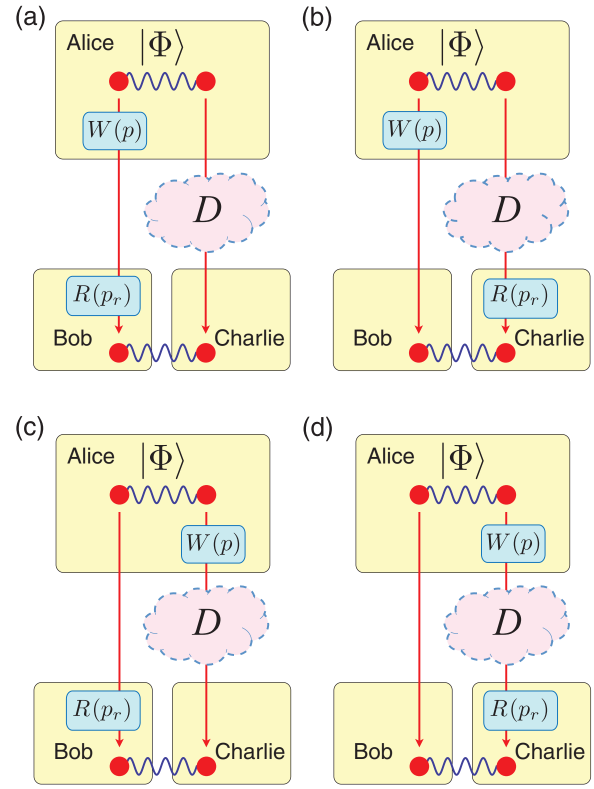

FIG. 2. (Color online) Alice prepares a pair of entangled qubits in state $|\Phi\rangle$ and sends them to Bob and Charlie. The qubit being sent to Charlie experiences decoherence D. The quantum state shared by Bob and Charlie, when $D = 0$, is the same in all cases. The degeneracy, however, is lifted due to decoherence-induced exchange symmetry breaking such that there exist two classes of quantum states shared by Bob and Charlie: ρ_B for (a) and (c) and ρ_C for (b) and (d).

collapses state $|\psi_1\rangle$ toward the opposite direction, the state becomes $|\psi_2\rangle = R(p_r)|\psi_1\rangle = \alpha\sqrt{1-p_r}\,|0\rangle + \beta\sqrt{1-p}\,|1\rangle$. If the reversing measurement strength $p_r = p$, state $|\psi_2\rangle$ becomes the original state $|\psi_0\rangle$, with the success probability of $1-p$. As discussed in Ref. [12], the weak measurement applied to an entangled state exhibits the exchange symmetry. Consider a two-qubit entangled state $|\Phi\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$ and weak measurement $W(p)$ is applied to one of the qubits (see Fig. 1). The reversing measurement $R(p_r)$ can be applied to the same qubit [local reversal; Fig. 1(a)] or the other qubit [nonlocal reversal; Fig. 1(b)]. If the reversing measurement strength $p_r = p$, both local and nonlocal reversal scenarios give the same result; that is, the original state $|\Phi\rangle$ is recovered, indicating the exchange symmetry of weak and reversing measurements.

Let us now investigate the exchange symmetry of weak measurement in an entangled state in the presence of decoherence. As shown in Fig. 2, Alice prepares a two-qubit entangled state $|\Phi\rangle$ and sends one qubit each to Bob and Charlie. The qubit being sent to Charlie undergoes a decoherence channel. Here, we consider amplitude damping decoherence in which the system qubit (S) couples to an environment qubit (E) by the following map: $|0\rangle_S|0\rangle_E \to |0\rangle_S|0\rangle_E$, $|1\rangle_S|0\rangle_E \to \sqrt{1-D}|1\rangle_S|0\rangle_E + \sqrt{D}|0\rangle_S|1\rangle_E$ [24,26,29], where D is the magnitude of the decoherence [16]. Note that, without weak and reversing measurements, the decoherence channel causes a nonunitary transformation of the initial two-qubit entangled state $|\Phi\rangle$ to the two-qubit mixed state ρ_D, and as a result, the degree of entanglement is reduced. Concurrence C, which quantifies the degree of entanglement [31], for state ρ_D is then calculated to be $C_D = 2|\alpha|\sqrt{(1-D)(1-|\alpha|^2)}$.

We now consider applying weak and reversing measurements similar to those in Fig. 1 to one of the qubits. Alice may choose to apply the weak measurement $W(p)$ to either of the two qubits before transmitting them to Bob and Charlie. The reversing measurement $R(p_r)$ is applied by Bob or Charlie. Thus, as depicted in Fig. 2, four distinct scenarios are possible for applying weak and reversing measurements. The reversing measurement strength we chose here is $p_r = p + D(1-p)$ [23,24,27]. When there is no decoherence, $D = 0$, the four scenarios shown in Fig. 2 result in the same final state; that is, there is exchange symmetry for weak and reversing measurements. With nonzero decoherence, however, the degeneracy is lifted due to decoherence-induced exchange symmetry breaking such that there exist two classes of quantum states shared by Bob and Charlie: ρ_B for Figs. 2(a) and 2(c) and ρ_C for Figs. 2(b) and 2(d). This is due to the fact that, while the location of Alice's weak measurement $W(p)$ does not affect the final state, whether Bob or Charlie performs the reversing measurement $R(p_r)$ causes the final state to differ. The concurrences for ρ_B and ρ_C are calculated to be

$$C_B = \frac{2(1-D)|\alpha|\sqrt{1-|\alpha|^2}}{1-D|\alpha|^2} \tag{1}$$

and

$$C_C = \frac{2|\alpha|\sqrt{1-|\alpha|^2}}{1+D(1-p)(1-|\alpha|^2)}, \tag{2}$$

respectively. Interestingly, C_B is independent of p, so for a given D, the concurrence is fixed regardless of the strength of the weak and reversing measurements. Also, $C_B < C_D$ unless $D < (2|\alpha|^2 - 1)/|\alpha|^4$, which means that doing the weak and reversing measurements makes the situation worse, reducing the amount of entanglement even further. Note that

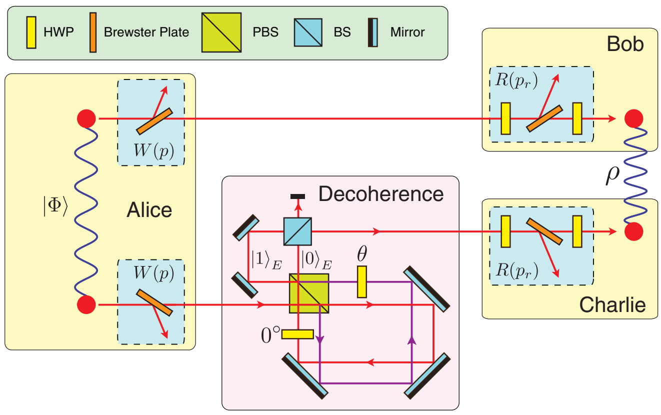

FIG. 3. (Color online) Alice prepares a pair of photonic polarization qubits in the maximally entangled state $|\Phi\rangle$ with $|\alpha| = 1/\sqrt{2}$ and sends them to Bob and Charlie. The qubit being sent to Charlie undergoes an amplitude-damping decoherence channel. The weak measurement $W(p)$ and the reversing measurement $R(p_r)$ are applied to one of the two qubits, resulting in the four possible scenarios depicted in Fig. 2. The quantum state ρ shared by Bob and Charlie is reconstructed by performing quantum state tomography.

if $|\alpha| < 1/\sqrt{2}$, C_B is always less than C_D. On the other hand, C_C approaches the concurrence of the initial entangled state, $2\,|\alpha|\,\sqrt{1-|\alpha|^2}$, with stronger weak measurement.

III. EXPERIMENT

To demonstrate the hitherto described effect of decoherence-induced exchange symmetry breaking, a pair of photonic polarization qubits and linear optical implementations of the weak measurement and the decoherence channel were used. The experimental setup is schematically shown in Fig. 3. A two-photon polarization state is generated by using the spontaneous parametric down-conversion process from a 6-mm-thick type-I β-BaB_2O_4 (BBO) crystal pumped with a 405-nm diode laser operating at 100 mW. Interference filters with a FWHM bandwidth of 5 nm are used to spectrally filter the photons. The two-photon maximally entangled state $|\Phi\rangle$ with $|\alpha| = 1/\sqrt{2}$ is prepared using the Shih-Alley quantum interferometry scheme [32]. Here, $|0\rangle$ and $|1\rangle$ refer to the horizontally and vertically polarized photons, respectively. The weak and reversing measurements for the polarization qubit can be implemented with half-wave plates (HWPs) and a Brewster plate, i.e., a glass plate oriented at the Brewster angle [20].

The amplitude-damping decoherence channel is implemented by using a displaced Sagnac interferometer, as shown in Fig. 3 [24,26,27,29]. The environment qubit is encoded in the path of the single photon. The horizontally polarized photon ($|0\rangle_S$) transmits the polarizing beam splitter (PBS) and always goes out into the $|0\rangle_E$ path, maintaining its polarization since it passes through HWP with an angle of 0°. On the other hand, the vertically polarized photon ($|1\rangle_S$) is reflected at the PBS and is then transformed into $\sqrt{D}|0\rangle_S + \sqrt{1-D}|1\rangle_S$ ($\sqrt{D} = \mathrm{Sin}2\theta$) after transmitting through the HWP with an angle of θ. When the state comes out from the PBS, the state has the form $\sqrt{D}|0\rangle_S|1\rangle_E + \sqrt{1-D}|1\rangle_S|0\rangle_E$. Finally, in order to trace out the environment system, we add a beam splitter (BS) to incoherently mix two environment qubits, $|0\rangle_E$ and $|1\rangle_E$. Note that the path length difference between the $|0\rangle_E$ and $|1\rangle_E$ modes is much larger than the coherence length ($\sim$140 μm) of the single photon.

First, for the initial entangled state $|\Phi\rangle$, we introduce the decoherence D gradually, without any weak and reversing measurements. The resulting quantum state ρ shared by Bob and Charlie is reconstructed via quantum state tomography [33]. The concurrence C calculated from this measurement is plotted in the inset of Fig. 4(a). This data set clearly shows that, due to the decoherence, the two-qubit state is losing entanglement.

IV. RESULTS AND ANALYSIS

We now examine the four scenarios depicted in Fig. 2 by introducing weak and reversing measurements. The decoherence was set at $D = 0.617$, and for each weak measurement strength p, the reversing measurement strength $p_r = p + D(1-p)$ was chosen. For each scenario in Fig. 2, the quantum state shared by Bob and Charlie was reconstructed for several different values of p. The concurrence values calculated from the reconstructed quantum states are shown in Fig. 4(a). The experimental data clearly show that, as discussed theoretically above, the four weak and reversing measurements scenarios in Fig. 2 exhibit two distinct quantum states due to nonzero decoherence. From the perspective of state evolution, this is due to the fact that, while the weak measurement may be applied to either of the two qubits without affecting the final state, the location of the reversing measurement affects the final state. In other words, the decoherence suppression effect of weak measurement emerges only when the reversing measurement is applied to the qubit experiencing the decoherence. This aspect of the result is clearly shown in Fig. 4(b), where the concurrence is plotted as a function of the linear entropy of the two-qubit state $S_L = 4(1 - \mathrm{tr}[\rho^2])/3$, which is a measure of the mixedness of quantum states [34]. With $D = 0.617$ and no weak and reversing measurements applied, the final state lies between the maximally entangled mixed state (MEMS) and the Werner state lines [35]. When the reversing measurement is applied to the qubit experiencing the decoherence, i.e., as done by Charlie, the stronger the weak measurement strength is, the higher the state purity and entanglement are. Note that, ideally, ρ_C is a MEMS if the initial state is maximally entangled, i.e., $|\Phi\rangle$ with $|\alpha| = 1/\sqrt{2}$. On the other hand, when the reversing measurement is done by Bob, changing the weak measurement strength has no effect on the state purity and entanglement. It is also interesting to note that decoherence-induced symmetry breaking is observed even though the decoherence only partially reduces the amount of entanglement in the two-qubit system.

V. CONCLUSION

In summary, we have shown in theory and in experiment that the exchange symmetry of weak measurement in an entangled state is broken when decoherence is introduced. Decoherence-induced exchange symmetry breaking of weak measurement occurs even though the qubits still share non-zero

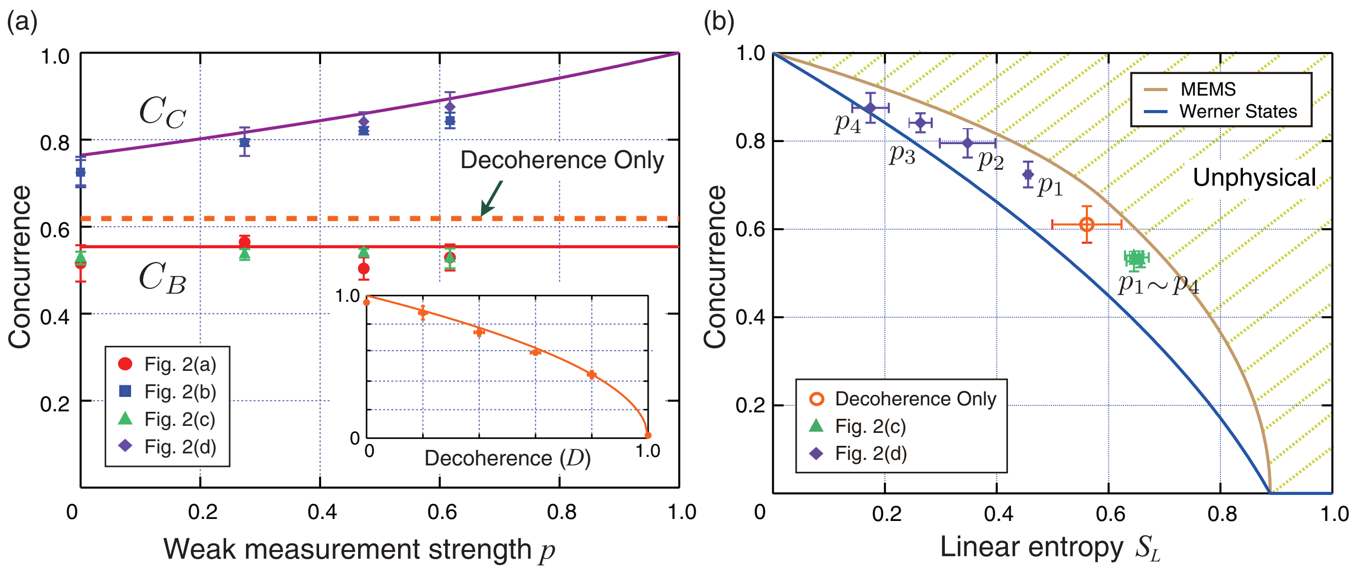

FIG. 4. (Color online) (a) For $D = 0.617$, concurrence vs the weak measurement strength p is plotted for the experimental scenario depicted in Fig. 2. For each p value, the reversing measurement strength $p_r = p + D(1 - p)$ was chosen [23,24,26,27]. Note that when $p = 0$, p_r is not zero but $p_r = D$. The experimental data clearly show that two distinct quantum states emerge due to nonzero decoherence, indicating decoherence-induced exchange symmetry breaking of weak measurement. The solid lines represent theoretical results for C_B and C_C. The inset shows concurrence vs decoherence without weak and reversing measurements. The solid line is a theoretical result. The vertical error bars represent the statistical error of ±1 standard deviation. The horizontal error bars in the inset reflect $\pm0.5^\circ$ angle errors for wave plates in the decoherence channel. (b) For different weak measurement strengths, $\{p_k\}_{k=1}^4 = \{0,0.274,0.473,0.617\}$, the concurrence and linear entropy S_L for ρ_B and ρ_C are plotted. The vertical and horizontal error bars represent a statistical error of ±1 standard deviation.

entanglement. Our results show that the decoherence suppression effect of the weak quantum measurements can be utilized only if the reversing measurement is applied to the qubit experiencing the decoherence. Hence, our results provide information on how and where to apply weak and reversing measurements to suppress decoherence in an entanglement distribution scenario between two distant parties [36]. Furthermore, in the context of the trade-off relations among information gain, state disturbance, and the reversibility of weak measurement [37], our results help to reestablish the trade-off relations when decoherence is introduced. Since weak measurements are local operations, our results could be extended to multipartite systems. We thus believe that our results shed light on multiqubit quantum state manipulation using general quantum operations in the presence of decoherence.

ACKNOWLEDGMENTS

This work was supported in part by the National Research Foundation of Korea (Grant No. 2013R1A2A1A01006029). H.-T.L. and J.-C.L. acknowledge support from the National Junior Research Fellowship (Grants No. 2012-000642 and No. 2012-000741, respectively).

[1] K. Gottfried and T.-M. Yan, *Quantum Mechanics: Fundamentals* (Springer, New York, 2003).
[2] R. Coldea, D. A. Tennant, E. M. Wheeler, E. Wawrzynska, D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl, and K. Kiefer, Science **327**, 177 (2010).
[3] P. Jarillo-Herrero, S. Sapmaz, C. Dekker, L. P. Kouwenhoven, and H. S. J. van der Zant, Nature (London) **429**, 389 (2004).
[4] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. **6**, 192 (2010).
[5] G. Toth and O. Gühne, Phys. Rev. Lett. **102**, 170503 (2009).
[6] P. Delaney, H. J. Choi, J. Ihm, S. G. Louie, and M. L. Cohen, Nature (London) **391**, 466 (1998).
[7] G. M. Luke *et al.*, Nature (London) **394**, 558 (1998).
[8] S. O. Demokritov, A. A. Serga, V. E. Demidov, B. Hillebrands, M. P. Kostylev, and B. A. Kalinikos, Nature (London) **426**, 159 (2003).
[9] F. Martin *et al.*, Science **315**, 629 (2007).
[10] G. Karpat and Z. Gedik, Opt. Commun. **282**, 4460 (2009).
[11] Y.-F. Huang, L. Peng, L. Li, B.-H. Liu, C.-F. Li, and G.-C. Guo, Phys. Rev. A **83**, 052105 (2011).
[12] A. C. Elitzur and S. Dolev, Phys. Rev. A **63**, 062109 (2001).
[13] J. D. Franson, Phys. Rev. A **45**, 3126 (1992).
[14] S.-Y. Baek, Y.-W. Cho, and Y.-H. Kim, Opt. Express **17**, 19241 (2009).
[15] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007).
[16] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[17] M. Koashi and M. Ueda, Phys. Rev. Lett. **82**, 2598 (1999).
[18] A. N. Korotkov and A. N. Jordan, Phys. Rev. Lett. **97**, 166805 (2006).

[19] N. Katz *et al.*, Phys. Rev. Lett. **101**, 200401 (2008).
[20] Y.-S. Kim, Y.-W. Cho, Y.-S. Ra, and Y.-H. Kim, Opt. Express **17**, 11978 (2009).
[21] X.-Y. Xu, J.-S. Xu, C.-F. Li, Y. Zou, and G.-C. Guo, Phys. Rev. A **83**, 010101(R) (2011).
[22] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[23] A. N. Korotkov and K. Keane, Phys. Rev. A **81**, 040103(R) (2010).
[24] J.-C. Lee, Y.-C. Jeong, Y.-S. Kim, and Y.-H. Kim, Opt. Express **19**, 16309 (2011).
[25] Q. Sun, M. Al-Amri, L. Davidovich, and M. S. Zubairy, Phys. Rev. A **82**, 052323 (2010).
[26] Y.-S. Kim, J.-C. Lee, O. Kwon, and Y.-H. Kim, Nat. Phys. **8**, 117 (2012).
[27] H.-T. Lim, J.-C. Lee, K.-H. Hong, and Y.-H. Kim, Opt. Express **22**, 19055 (2014).
[28] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).
[29] M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, Science **316**, 579 (2007).
[30] T. Yu and J. H. Eberly, Science **323**, 598 (2009).
[31] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[32] Y. H. Shih and C. O. Alley, Phys. Rev. Lett. **61**, 2921 (1988).
[33] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A **64**, 052312 (2001).
[34] N. A. Peters, T.-C. Wei, and P. G. Kwiat, Phys. Rev. A **70**, 052309 (2004).
[35] W. J. Munro, D. F. V. James, A. G. White, and P. G. Kwiat, Phys. Rev. A **64**, 030302(R) (2001).
[36] J.-C. Lee, H.-T. Lim, K.-H. Hong, Y.-C. Jeong, M. S. Kim, and Y.-H. Kim, Nat. Commun. **5**, 4522 (2014).
[37] H.-T. Lim, Y.-S. Ra, K.-H. Hong, S.-W. Lee, and Y.-H. Kim, Phys. Rev. Lett. **113**, 020504 (2014).