

# Entangling two separate photonic ququarts using linear optical elements

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We report an experimental demonstration of entangling two separable photonic ququarts using linear optical elements. Each ququart is implemented by using the dichotomic spatial and polarization modes of a single photon. By interfering two independent ququarts at a polarizing beam splitter, we experimentally generate entangled ququarts.

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## I. INTRODUCTION

In quantum information, the basic unit of quantum information is the quantum bit or qubit. The qubit can be implemented by any two-dimensional quantum state, such as an electron spin, a photon polarization, two discrete energy levels of an atom, and so on. The intrinsic quantum nature of the qubit originates from two distinct properties to the classical bit: quantum superposition for a single qubit and quantum entanglement for multiple qubits. Quantum information harnesses these properties so as to overcome the limits of classical information [1].

The use of a higher dimensional quantum system or qudits for  $d$ -dimensional quantum states, instead of qubits, has some fundamental and practical advantages in quantum information science [2–4]. For example, the qudits are more robust against optical noise in the transmission channel than the qubits. Therefore, the qudits are advantageous not only for a long distance transmission but also for an increase of the secure key rate for quantum cryptography [5,6]. In addition, it has been proven that using qudits instead of qubits can increase the efficiency of the Bell-state measurement for long-distance quantum teleportation and the efficiency of quantum gates and quantum information protocols [7–9].

In the quantum optical approach to quantum information, qudits can be implemented with a high-dimensional degree of freedom of a photon, such as orbital-angular momentum (OAM), multiple paths, multiple time bins, and so on [10–14]. The hybrid of different modes, for example, polarization-paths, polarization-spectral modes, and polarization-OAM, provides another way to implement a qudit [15–17].

While generating a qudit itself is of importance, entangling those qudits is another crucial task for quantum information science with qudits. Recently, entanglement of two photonic qudits has been demonstrated by utilizing the high dimensional bases, such as OAM, transverse momentum-position, time-of-arrival, etc. [18–23]. It is worth noting that the entanglement in these experiments relies on the intrinsic correlation nature of photon pairs from spontaneous parametric down-conversion. However, the scalable qudits are necessary for many quantum information applications. To this end, it is better to separately implement the preparation of individual qudits and the entangling operation. The entanglement generation

schemes utilizing the intrinsic correlation nature of photon pairs, however, cannot be considered as such processes. Note that there have been some theoretical studies to realize the scalable entanglement generation of qudits [24,25]. In Refs. [24,25], the authors explicitly analyzed the generation of entanglement of four-dimensional qudits or ququarts. Each ququart is prepared with the polarization and spectral modes of a biphoton. The entangling operations can be achieved by four-photon interference either at an ordinary, polarizing, or dichroic beam splitter. Although the scheme is reliable and feasible, it is difficult to be implemented as it requires four-photon interference.

In this paper, we report an experimental generation of two-ququart entangled states using linear optical elements. Each ququart is implemented by using the dichotomic spatial and polarization modes of a single photon. After two separable ququarts interfere at a polarizing beam splitter (PBS), we postselect the case that one-ququart resides at each output of the PBS, resulting in a two-ququart entangled state. Also, we experimentally show that it is possible to generate various two-ququart entangled states.

## II. THEORY

Let us first consider the scheme for single-photon ququart generation shown in Fig. 1(a). A half-wave plate (HWP) combined with a PBS determines the probability amplitudes of the spatial modes,  $i_1$  and  $i_2$ . The spatial phase  $\phi_i$  of a single-photon ququart state is determined by scanning a mirror  $M_i$ . The polarization state at each spatial mode is controlled by wave plates (WPs) consisting of a combination of half- and quarter-wave plates.

The four orthonormal states, constructing the ququart bases, can be defined as follows:

$$\begin{aligned} |H, i_1\rangle &\equiv |0\rangle_{i_1} & |V, i_1\rangle &\equiv |1\rangle_{i_1} \\ |H, i_2\rangle &\equiv |2\rangle_{i_2} & |V, i_2\rangle &\equiv |3\rangle_{i_2} \end{aligned} \quad (1)$$

where  $H$  and  $V$  refer to the horizontal and vertical polarization of the photon, respectively. The subscript  $i$  refers to the input modes of the PBS,  $a$  and  $b$ , and the subscripts 1 and 2 denote the spatial mode of each input. Then, an arbitrary single-photon ququart at each input of PBS can be written as

$$|\psi\rangle_i = \alpha_i |0\rangle_i + \beta_i |1\rangle_i + \gamma_i |2\rangle_i + \delta_i |3\rangle_i, \quad (2)$$

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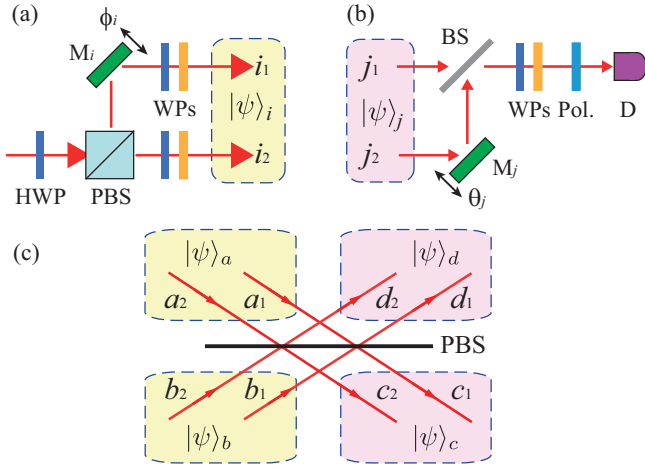


FIG. 1. (Color online) Scheme for (a) preparation and (b) measurement of a single-photon ququart.  $i_1$  and  $i_2$  ( $j_1$  and  $j_2$ ) are spatial modes in preparation (measurement).  $\phi_i$  ( $\theta_j$ ) refers to the relative phase between  $i_1$  and  $i_2$  ( $j_1$  and  $j_2$ ). HWP: half-wave plate, PBS: polarizing beam splitter, M: mirror, WPs: half- and quarter-wave plates, BS: beam splitter, Pol.: polarizer, and D: single-photon detector. (c) The entangling operation of two single-photon ququarts using a PBS. The PBS has input ports  $a$  and  $b$ , each subdividing into  $i_1$  and  $i_2$ , and output ports  $c$  and  $d$ , each into  $j_1$  and  $j_2$ . Two separable ququarts enter the PBS via different input ports. When each ququart exits through different output ports, the resultant ququarts become entangled.

where the  $\alpha_i, \beta_i, \gamma_i$ , and  $\delta_i$  denote the complex probability amplitudes that satisfy  $|\alpha_i|^2 + |\beta_i|^2 + |\gamma_i|^2 + |\delta_i|^2 = 1$ . Note that the four amplitudes  $\alpha_i, \beta_i, \gamma_i$ , and  $\delta_i$  are controlled by a HWP and WPs in Fig. 1(a), and thus we can generate an arbitrary single-photon ququart  $|\psi\rangle_i$ .

For analyzing the generated ququarts, we implement the projection measurement scheme shown in Fig. 1(b). Since the basis of a ququart consists of spatial modes and polarization modes, the ququarts are measured at each degree of freedom, respectively. First, the spatial modes  $j_1$  and  $j_2$  are measured by combining the two spatial modes at a beam splitter (BS). The phase  $\theta_j$  between two spatial modes can be scanned by a mirror  $M_j$ . Note that  $\theta_j$  should be determined in terms of the relative phase with respect to  $\phi_i$  since the ququart generation and measurement schemes together compose a single-photon interferometer [26]. The polarization modes are measured by a set of wave plates and a polarizer [27,28].

The two-ququart entangled state can be generated by interfering two ququarts ( $|\psi\rangle_a$  at mode  $a$  and  $|\psi\rangle_b$  at mode  $b$ ) at a PBS, see Fig. 1(c). At the input of the PBS, the two ququarts are separable:

$$|\psi\rangle_{\text{in}} = |\psi\rangle_a \otimes |\psi\rangle_b. \quad (3)$$

Then,  $|\psi\rangle_{\text{in}}$  is transformed at the PBS by the following relations:

$$\begin{aligned} |H, a_s\rangle &\rightarrow |H, c_s\rangle, & |V, a_s\rangle &\rightarrow |V, d_s\rangle, \\ |H, b_s\rangle &\rightarrow |H, d_s\rangle, & |V, b_s\rangle &\rightarrow |V, c_s\rangle, \end{aligned} \quad (4)$$

where  $c$  and  $d$  are the output modes of the PBS and the subscript  $s \in \{1, 2\}$  is the upper or lower path in each output mode.

If we only consider the case that each output of the PBS has one photon, the postselected two ququart state at the outputs of the PBS is

$$\begin{aligned} |\psi\rangle_{cd} = & \alpha_a \alpha_b |0\rangle_c |0\rangle_d + \alpha_a \gamma_b |0\rangle_c |2\rangle_d \\ & + \beta_a \beta_b |1\rangle_d |1\rangle_c + \beta_a \delta_b |1\rangle_d |3\rangle_c \\ & + \gamma_a \alpha_b |2\rangle_c |0\rangle_d + \gamma_a \gamma_b |2\rangle_c |2\rangle_d \\ & + \delta_a \beta_b |3\rangle_d |1\rangle_c + \delta_a \delta_b |3\rangle_d |3\rangle_c. \end{aligned} \quad (5)$$

In general, Eq. (5) cannot be represented by  $|\psi\rangle_c \otimes |\psi\rangle_d$  and thus it is a two-ququart entangled state. Furthermore, one can generate various forms of entanglement states by changing the coefficients  $\alpha_i, \beta_i, \gamma_i$ , and  $\delta_i$  of two input states of Eq. (3).

### III. EXPERIMENT

The experimental setup for entangling two ququarts is shown in Fig. 2. The setup can be divided into four parts: (a) generation of a photon pair, (b) preparation of two separable ququarts, (c) entangling operation of the two ququarts, and (d) analysis of the generated states. First, in Fig. 2(a), a pair of photons is generated in a 6-mm-thick type-I BBO ( $\beta$ -BaB<sub>2</sub>O<sub>4</sub>) crystal by the spontaneous parametric down-conversion (SPDC) process pumped by a diode laser (central wavelength of 405 nm and average power of 100 mW) [29,30]. The generated photons have an 810-nm central wavelength, and they are filtered by bandpass filters with a full width at half-maximum bandwidth of 40 nm. Coincidence counts by the photons amount to 20 kHz, and each photon will be used to encode a ququart.

The preparation of each single-photon ququart is shown in Fig. 2(b). After a single photon passes through a fiber polarization controller (FPC), a PBS, a mirror (M1/M2), and two HWPs, a single-photon ququart can be prepared: the FPC changes transmission and reflection ratios at the PBS, which functions as the HWP before the PBS in Fig. 1(a); M1/M2 controls the phase between the two spatial modes, and the HWPs control polarization. Here, we omit QWPs in Fig. 1(a), and therefore, the relative phase between  $\alpha_i$  and  $\beta_i$ , and similarly the phase between  $\gamma_i$  and  $\delta_i$ , cannot be controlled. Nevertheless, one can still generate various ququarts, as will be described in the next section.

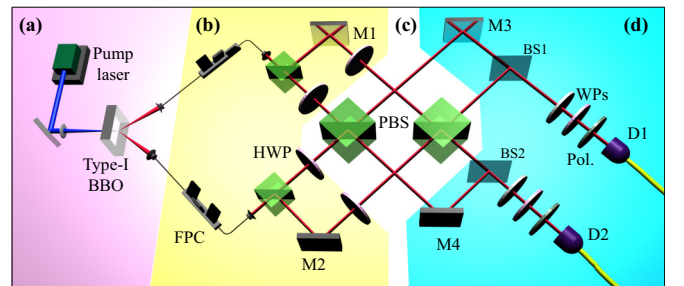


FIG. 2. (Color online) Experiment setup. (a) SPDC process for a photon pair generation, (b) preparation of two independent single-photon ququarts, (c) entangling operation, and (d) analysis of the generated state using quantum state tomography. FPC: fiber polarization controller.

TABLE I. The projection measurements for two-ququarts quantum state tomography.

Projection measurement	Mode $c$		Mode $d$	
	Polarization	Spatial	Polarization	Spatial
1	$ H\rangle$	$ c_1\rangle$	$ H\rangle$	$ d_1\rangle$
2	$ V\rangle$	$ c_2\rangle$	$ V\rangle$	$ d_2\rangle$
3	$ D\rangle$	$ c_d\rangle$	$ A\rangle$	$ d_d\rangle$
4	$ R\rangle$	$ c_r\rangle$	$ L\rangle$	$ d_r\rangle$

Then, two separable ququarts interfere at the PBS, as shown in Fig. 2(c). At the outputs of the PBS, we select the cases that a single photon resides at each output mode. With this postselection, the generated states are entangled as shown in Eq. (5).

The final state is analyzed by quantum state tomography (QST) [27]. Figure 2(d) shows the experimental setup for QST. For complete analysis of two-ququart states, both of the polarization (PM) and spatial modes (SM) should be measured. Table I presents required projection measurements for QST. Here, the polarization and spatial states in Table I are defined as

$$\begin{Bmatrix} |D\rangle \\ |A\rangle \end{Bmatrix} = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle), \quad \begin{Bmatrix} |R\rangle \\ |L\rangle \end{Bmatrix} = \frac{1}{\sqrt{2}}(|H\rangle \pm i|V\rangle), \quad (6)$$

and

$$|j_d\rangle = \frac{1}{\sqrt{2}}(|j_1\rangle + |j_2\rangle), \quad |j_r\rangle = \frac{1}{\sqrt{2}}(|j_1\rangle + i|j_2\rangle), \quad (7)$$

where  $j$  refers to the output modes of the PBS,  $c$  and  $d$ . We perform projective measurements on polarization modes by using a HWP, a QWP, and a Pol, see Fig. 2. To implement projective measurements on  $|j_d\rangle$  and  $|j_r\rangle$ , we combine different spatial modes  $j_1$  and  $j_2$  using BS1/BS2 and adjust the phase difference between the modes by moving M3/M4; on the other hand, for projecting on  $|j_1\rangle$  ( $|j_2\rangle$ ), we block path  $j_2$  ( $j_1$ ). Note that at D2 the projection states on polarization are changed from  $|D\rangle$  and  $|R\rangle$  to  $|A\rangle$  and  $|L\rangle$  because a  $\pi$  phase shift takes place between  $|H\rangle$  and  $|V\rangle$  at the BS output. All combinations of projective measurements in polarization and spatial modes are required for complete characterization of two ququarts; therefore, 256 projective measurements in overall are performed.

The experimental setup comprising the preparation of ququarts, entangling operation, and analysis of generated ququarts becomes interferometers, which require phase stabilities between different paths. However, in the experiment, phase stabilities are maintained only for a few seconds. To circumvent this problem, we constantly scan mirrors M3 and M4, which modifies the phase difference  $\phi_i - \theta_j$  [26], and we postselect data only when desired phase differences are made.

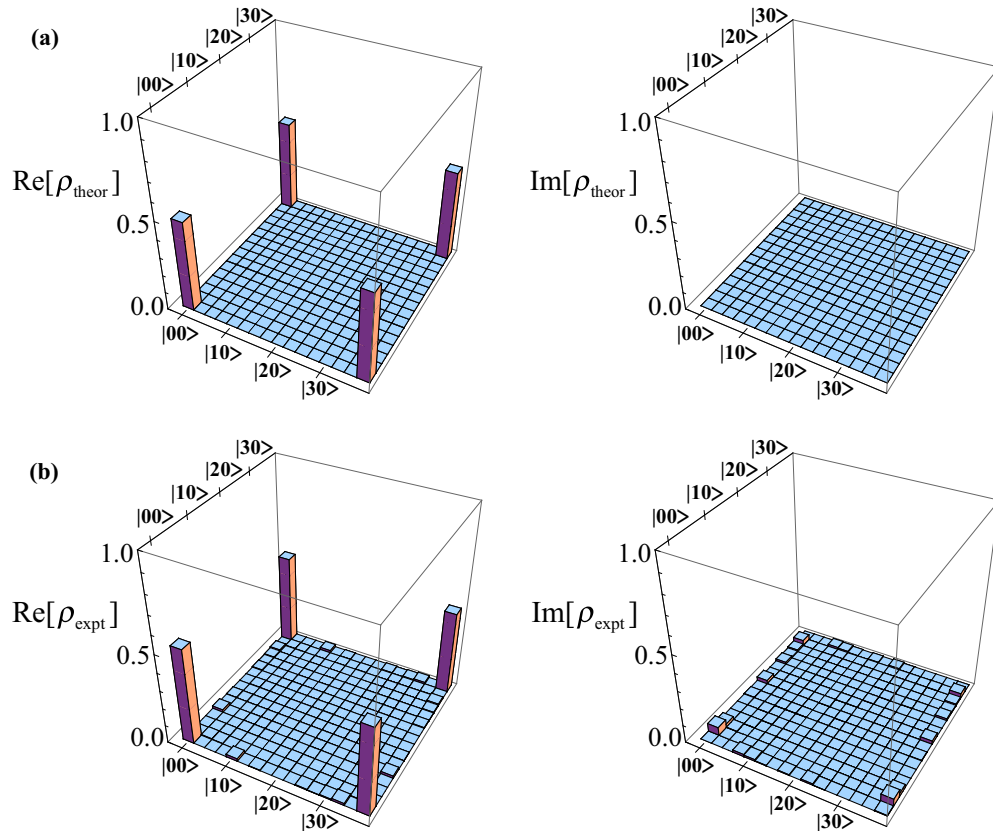


FIG. 3. (Color online) (a) The theoretically expected  $\rho_{\text{theor}}$  and (b) the experimentally reconstructed  $\rho_{\text{expt}}$  density matrices of Eq. (9). The calculated fidelity is  $F = 0.987$ , negativity is  $N = 0.519$ , and the purity is  $P = 1.000$ . The axis represents 16 orthonormal basis states of two ququarts in an increasing order:  $|00\rangle, |01\rangle, \dots, |33\rangle$ .

#### IV. RESULTS AND ANALYSIS

Let us present two cases for entanglement generation of two ququarts. First, we perform the simplest case that two out of four coefficients in Eq. (2) are zero. To this end, we control a FPC, see Fig. 2(b), so as to have the same probability at each spatial mode. The HWP angle in each spatial mode is set to zero to ensure each spatial mode has either horizontal or vertical polarization. With this condition, we can make  $\beta_i = \gamma_i = 0$ , and prepare the input states of

$$\begin{aligned} |\psi\rangle_a &= \frac{1}{\sqrt{2}}(|0\rangle_a + |3\rangle_a), \\ |\psi\rangle_b &= \frac{1}{\sqrt{2}}(|0\rangle_b + |3\rangle_b). \end{aligned} \quad (8)$$

After they interfere at a PBS, the state that each output of the PBS has one photon is given as

$$|\psi\rangle_{\text{out}} = \frac{1}{\sqrt{2}}(|0\rangle_c|0\rangle_d + |3\rangle_c|3\rangle_d), \quad (9)$$

which is clearly an entangled state. Note that Eq. (9) shows how two input quantum states, each populated only in two modes, can generate a simplest form of an entangled state (i.e., similar to a Bell state).

Figures 3(a) and 3(b) show the theoretically expected density matrix  $\rho_{\text{theor}}$  and the experimentally reconstructed density matrix  $\rho_{\text{expt}}$  for Eq. (9), respectively. To evaluate the experimentally obtained state, we calculate the followings quantities: fidelity, negativity, and purity. First, fidelity,  $F =$

$(\text{Tr} \sqrt{\sqrt{\rho_{\text{theor}}} \rho_{\text{expt}} \sqrt{\rho_{\text{theor}}}})^2$ , between the experimental state  $\rho_{\text{expt}}$  and the theoretical state  $\rho_{\text{theor}}$  is  $F = 0.987$ , which shows that our experimental ququart entangled state is very close to the theoretical one. Second, negativity [31],  $N = (||\rho_{\text{expt}}^{\text{PT}}|| - 1)/2$ , is employed to verify whether the experimental state is entangled or not. Here,  $||\rho_{\text{expt}}^{\text{PT}}||$  is the trace norm of a partial transposition of  $\rho_{\text{expt}}$ . One can certify that there is nonzero entanglement if  $N > 0$ . The negativity of the experimental state is  $N = 0.519$  [cf. for ideal state in Eq. (9),  $N = 0.5$ ], which clearly shows that the final state is entangled. Finally, purity  $\text{Tr}(\rho_{\text{expt}}^2)$  of the state is calculated to be  $P = 1.000$ , which shows that the state is highly pure.

In order to implement a more complicated state, we populate all the four modes only in one of the input quantum states as

$$\begin{aligned} |\psi\rangle_a &= \frac{1}{\sqrt{2}}(|0\rangle_a + e^{i\phi}|3\rangle_a), \\ |\psi\rangle_b &= \frac{1}{2}(|0\rangle_b + |1\rangle_b + |2\rangle_b + |3\rangle_b). \end{aligned} \quad (10)$$

The relative phase  $\phi$  between two different modes is set to be  $\pi/4$ . After interfering them at a PBS, the output state becomes

$$|\psi\rangle_{\text{out}} = \frac{1}{2}(|0\rangle_c|0\rangle_d + |2\rangle_c|0\rangle_d + e^{i\phi}|3\rangle_c|1\rangle_d + e^{i\phi}|3\rangle_c|3\rangle_d). \quad (11)$$

The theoretically expected  $\rho_{\text{theor}}$  and experimentally reconstructed  $\rho_{\text{expt}}$  density matrices are shown in Figs. 4(a) and 4(b), respectively. We obtained the high fidelity,  $F = 0.956$ , and

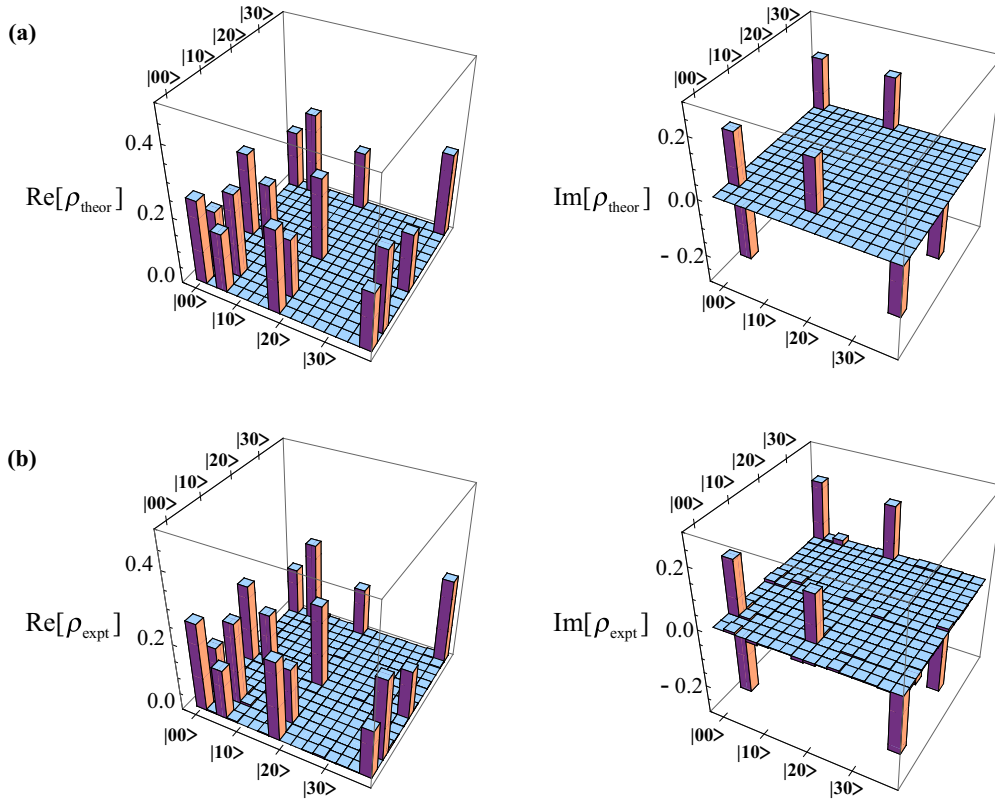


FIG. 4. (Color online) (a) The theoretically expected  $\rho_{\text{theor}}$  and (b) the experimentally reconstructed  $\rho_{\text{expt}}$  density matrices of Eq. (11). The calculated fidelity is  $F = 0.956$ , negativity is  $N = 0.496$ , and the purity is  $P = 0.925$ .



this result shows that the experimental state is close to the theoretical one. The negativity and purity are  $N = 0.496$  [cf. for ideal state in Eq. (11),  $N = 0.5$ ] and  $P = 0.925$ , respectively. With these results, we can verify that the final state is a highly-pure ququart entangled state.

## V. CONCLUSION

In conclusion, we have demonstrated the generation of entanglement of two independent ququarts using linear optical elements. By interfering two separable ququarts, we can generate various forms of the two-ququart entangled state. The entanglement between the ququarts is generated by applying an entangling operation on separable ququarts rather than exploiting the intrinsic correlations in the SPDC process used

in Refs. [18–23]. We believe that our scheme to generate entangled states of two qudits will be useful for a high-dimensional entanglement source for quantum information science, and the entangling operation used can be adopted as a nonlocal operation for implementing quantum circuits.

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