Remote preparation of three-photon entangled states via single-photon measurement

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Remote state preparation (RSP) provides an indirect way of transferring quantum information based on the nonlocal effect of quantum measurement. Although RSP has been demonstrated in recent years to remotely prepare multiphoton states, quantum measurement on the same number of additional photons was required, i.e., to prepare N-photon states via RSP, quantum measurement on the other N-photons was required, hence significantly limiting practicality and applicability of RSP. Here we report an experimental demonstration of remote preparation of three-photon entangled states by measuring only a single-photon entangled with the three photons. We further generalize our protocol to prepare multiphoton entangled states with arbitrary photon number and purity and to prepare a genuinely three-partite entangled state, both via single-photon measurement. As our RSP scheme relies on the nonlinearity induced by single-photon measurement, it enables quantum state engineering of multiphoton entangled states beyond the linear optical limit. Our results are expected to have significant impacts on quantum metrology and quantum information processing.

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I. INTRODUCTION

Entanglement, once conceived as the "weirdness" of quantum mechanics [1], is now at the heart of quantum technologies, such as quantum communication [2,3], quantum computing [4–6], and quantum metrology [7,8]. Communication of quantum information based on the nonlocal nature of quantum measurement on entangled quantum systems can be categorized into two: quantum teleportation [9–11] and remote state preparation (RSP) [12,13]. In quantum teleportation, Alice can transfer an *unknown* qubit to Bob by sending him two bits of classical information resulting from the Bell state measurement [11], assuming Alice and Bob already share a pair of maximally entangled qubits. In RSP, Alice and Bob are still assumed to share a pair of maximally entangled qubits, but Alice can transfer a *known* qubit to Bob by sending him only one bit of information from her measurement. Moreover, as RSP does not require complete Bell state measurement, it can be more readily scaled up for larger quantum systems.

In photonic systems, RSP has recently been demonstrated for single photons [14–16], two photons [17,18], and three photons [19]. In these RSP schemes, however, quantum measurement on the same number of additional photons was required, i.e., to prepare an N-photon state at Bob, N-photon quantum measurement is required at Alice, hence significantly limiting practicality and applicability of RSP. However, RSP does not necessarily require Alice to measure the same number of photons as the remotely prepared quantum state on Bob. In fact, if Alice and Bob share an entangled state consisting of a single photon and multiple photons, single-photon measurement at Alice would be sufficient to remotely prepare a multiphoton state at Bob.

In this work, we demonstrate remote preparation of various three-photon entangled states via single-photon measurement by preparing entanglement between a single photon and three single photons. Alice's measurement on the single photon can transfer quantum information to a three-photon entangled state at Bob. We also generalize our RSP protocol to prepare multiphoton entangled states of arbitrary photon number and purity and to prepare a genuinely three-partite entangled state, both via single-photon measurement. Our RSP protocol extends the capability of multiphoton state engineering beyond the linear optical limit via the nonlinearity induced by the single-photon measurement [4,5,20], allowing us to prepare various multiphoton states required for quantum metrology [21–24] and for fundamental studies in quantum optics [25,26].

II. THE PROTOCOL

Remote preparation of a three-photon entangled state by single-photon measurement requires Alice and Bob to share an entangled state between a single photon and three single photons of the form,

$$|\Phi\rangle_{AB} = \frac{|1_H,0_V\rangle_A|1_H,2_V\rangle_B + |0_H,1_V\rangle_A|2_H,1_V\rangle_B}{\sqrt{2}}, \quad (1)$$

where subscripts A, B, H, and V refer to Alice, Bob, horizontal polarization, and vertical polarization, respectively. Alice measures her single photon in the basis $\{|\phi\rangle_A,|\phi_\perp\rangle_A\}$, where $|\phi\rangle_A = \beta|1_H,0_V\rangle_A + \alpha e^{i\theta}|0_H,1_V\rangle_A$ with real parameters α,β, and θ. Note that ${}_A\langle\phi_\perp|\phi\rangle_A = 0$. When Alice's single photon is measured in the basis $|\phi\rangle_A$, Bob's three-photon state is then projected onto

$$|\psi\rangle_B = \alpha|2_H,1_V\rangle_B + \beta e^{i\theta}|1_H,2_V\rangle_B. \quad (2)$$

Therefore, the single-photon measurement at Alice can remotely prepare the desired three-photon entangled state at Bob.

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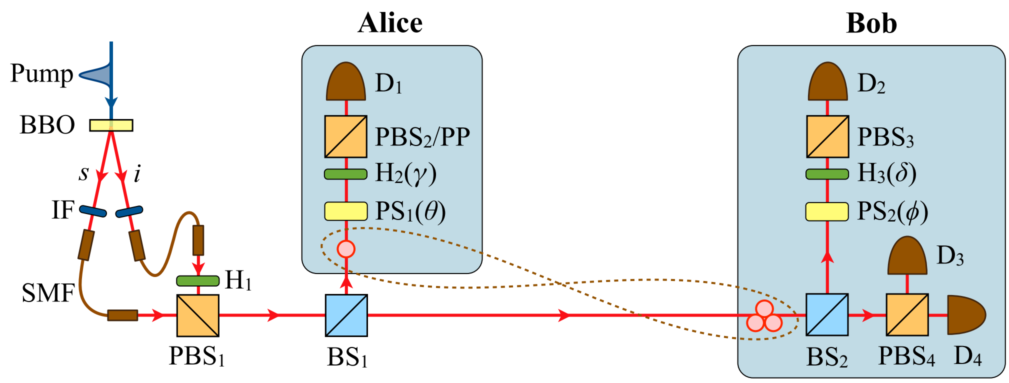

FIG. 1. Experimental setup. At the input of a nonpolarizing beam splitter BS_1 (T, 80%, R, 20%), the quantum state of four photons is $|2_H,2_V\rangle$, and when a single photon is reflected and the other three photons are transmitted, Alice and Bob share the entangled state in Eq. (1). Alice measures the single photon by projection onto $\cos 2\gamma|1_H,0_V\rangle_A + e^{i\theta}\sin 2\gamma|0_H,1_V\rangle_A$ using a phase shifter $PS_1(\theta)$, a half-wave plate $H_2(\gamma)$, a polarizing beam splitter PBS_2, and a single-photon avalanche detector D_1 (Perkin-Elmer SPCM-AQRH-13). As a consequence, the three-photon entangled state in Eq. (2) is prepared at Bob, and he measures the three photons by projection onto $\cos 2\delta|2_H,1_V\rangle_B + e^{i\phi}\sin 2\delta|1_H,2_V\rangle_B$ using BS_2, $PS_2(\phi)$, $H_3(\delta)$, PBS_3, PBS_4, and coincidence detection on D_2, D_3, and D_4. RSP of a three-photon mixed state [Eq. (9)] requires the use of a partial polarizer (PP) instead of PBS_2 at Alice [14]. The phase shifters $PS_1(\theta)$ and $PS_2(\phi)$ are implemented by rotating a half-wave plate between two quarter-wave plates set at the angle of $\pi/4$. BBO, β-BaB_2O_4 crystal; s, signal; i, idler; IF, interference filter; SMF, single-mode fiber.

III. ENTANGLEMENT BETWEEN A SINGLE PHOTON AND THREE PHOTONS

To prepare entanglement between a single photon and three photons, we employ the experimental setup shown in Fig. 1. A femtosecond pulse laser (duration of 100 fs and average power of 165 mW) impinges on a BBO (β-BaB_2O_4, thickness of 2 mm) crystal, which generates two horizontally polarized photons at each of signal s and idler i modes; the photon-pair generation probability is set to low (0.017) to suppress contaminations from higher order photon-pair generations. To eliminate spectral and spatial correlations of the photons, interference filters (IFs, full width at half maximum bandwidth of 3 nm centered at 780 nm) and single-mode fibers (SMFs) are used. By using a half-wave plate H_1 at the angle of $\pi/4$, the polarization state of idler photons is rotated to vertical polarization just prior to the polarizing beam splitter (PBS_1). All the photons are made to arrive at PBS_1 simultaneously and the quantum state of the photons at the output of PBS_1 is $|2_H,2_V\rangle$. At a nonpolarizing beam splitter BS_1, when a single photon is reflected to Alice and the other three photons are directed to Bob, the single photon and the three photons are in the entangled state in Eq. (1). Such partition of the four photons is assured by detecting fourfold coincidence counts on D_1–D_4.

To verify entanglement between the single photon and the three photons, we test the Clauser-Horne-Shimony-Holt (CHSH) inequality [27], which provides a strict entanglement test on bipartite systems [28]. The Hilbert space for the quantum state under the test has the dimension $2 \otimes 2$ because, for the single photon, the basis is $\{|1_H,0_V\rangle,|0_H,1_V\rangle\}$ and, for the three photons, the basis is $\{|2_H,1_V\rangle,|1_H,2_V\rangle\}$. The CHSH parameter is then defined as

$$S_{\mathrm{CHSH}} = |-\langle\hat{\mu}^{\mathrm{s}}\hat{\mu}^{\mathrm{t}}\rangle + \langle\hat{\mu}^{\mathrm{s}}\hat{\pi}^{\mathrm{t}}\rangle + \langle\hat{\pi}^{\mathrm{s}}\hat{\mu}^{\mathrm{t}}\rangle + \langle\hat{\pi}^{\mathrm{s}}\hat{\pi}^{\mathrm{t}}\rangle|, \quad (3)$$

where $\hat{\mu}^{\mathrm{s}} = \frac{1}{\sqrt{2}}(\hat{\sigma}_z^{\mathrm{s}} + \hat{\sigma}_x^{\mathrm{s}})$ and $\hat{\pi}^{\mathrm{s}} = \frac{1}{\sqrt{2}}(\hat{\sigma}_z^{\mathrm{s}} - \hat{\sigma}_x^{\mathrm{s}})$ are measurements on the single photon, and $\hat{\mu}^{\mathrm{t}} = \hat{\sigma}_z^{\mathrm{t}}$ and $\hat{\pi}^{\mathrm{t}} = \hat{\sigma}_x^{\mathrm{t}}$ are measurements on the three photons. The relevant operators are defined as

$$\begin{aligned}
\hat{\sigma}_z^{\mathrm{s}} &= |1_H,0_V\rangle\langle 1_H,0_V| - |0_H,1_V\rangle\langle 0_H,1_V|,\\
\hat{\sigma}_x^{\mathrm{s}} &= |1_H,0_V\rangle\langle 0_H,1_V| + |0_H,1_V\rangle\langle 1_H,0_V|,\\
\hat{\sigma}_z^{\mathrm{t}} &= |2_H,1_V\rangle\langle 2_H,1_V| - |1_H,2_V\rangle\langle 1_H,2_V|,\\
\hat{\sigma}_x^{\mathrm{t}} &= |2_H,1_V\rangle\langle 1_H,2_V| + |1_H,2_V\rangle\langle 2_H,1_V|, \quad (4)
\end{aligned}$$

and the correlation values $\langle\hat{\lambda}^{\mathrm{s}}\hat{\lambda}^{\mathrm{t}}\rangle$ can be obtained from joint measurements on the single photon $\hat{\lambda}^{\mathrm{s}}$ and on the three photons $\hat{\lambda}^{\mathrm{t}}$,

$$\langle\hat{\lambda}^{\mathrm{s}}\hat{\lambda}^{\mathrm{t}}\rangle = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}}, \quad (5)$$

where N_{ij} is the coincidence-count rate by $i \in (+,-)$ at $\hat{\lambda}^{\mathrm{s}}$ and $j \in (+,-)$ at $\hat{\lambda}^{\mathrm{t}}$. For the single-photon measurement, PS_1 is removed and PBS_2 is used. The outcomes $+$ and $-$ of $\hat{\lambda}^{\mathrm{s}} = \hat{\mu}^{\mathrm{s}}$ correspond to a click on D_1 when the angle γ of H_2 is $\pi/16$ and $5\pi/16$, respectively. Similarly, $+$ and $-$ outcomes of $\hat{\lambda}^{\mathrm{s}} = \hat{\pi}^{\mathrm{s}}$ are obtained by setting the angle γ of H_2 with $3\pi/16$ and $7\pi/16$, respectively. For the three-photon measurement, PS_2 is removed, and coincidence clicks on D_2, D_3, and D_4 are recorded: The outcomes $(+,-)$ are obtained by setting the angle δ of H_3 at $(0,\pi/4)$ for $\hat{\lambda}^{\mathrm{t}} = \hat{\mu}^{\mathrm{t}}$ and at $(\pi/8,3\pi/8)$ for $\hat{\lambda}^{\mathrm{t}} = \hat{\pi}^{\mathrm{t}}$. We obtained S_{CHSH} violation more than 7 standard deviations, summarized in Table I.

IV. REMOTE PREPARATION OF THREE-PHOTON ENTANGLED STATES

Using the entanglement between the single photon and the three photons, we carry out RSP of a three-photon entangled state with an adjustable phase θ, i.e.,

$$|\psi\rangle_B^{\mathrm{phase}} = \frac{1}{\sqrt{2}}(|2_H,1_V\rangle_B + e^{i\theta}|1_H,2_V\rangle_B), \quad (6)$$

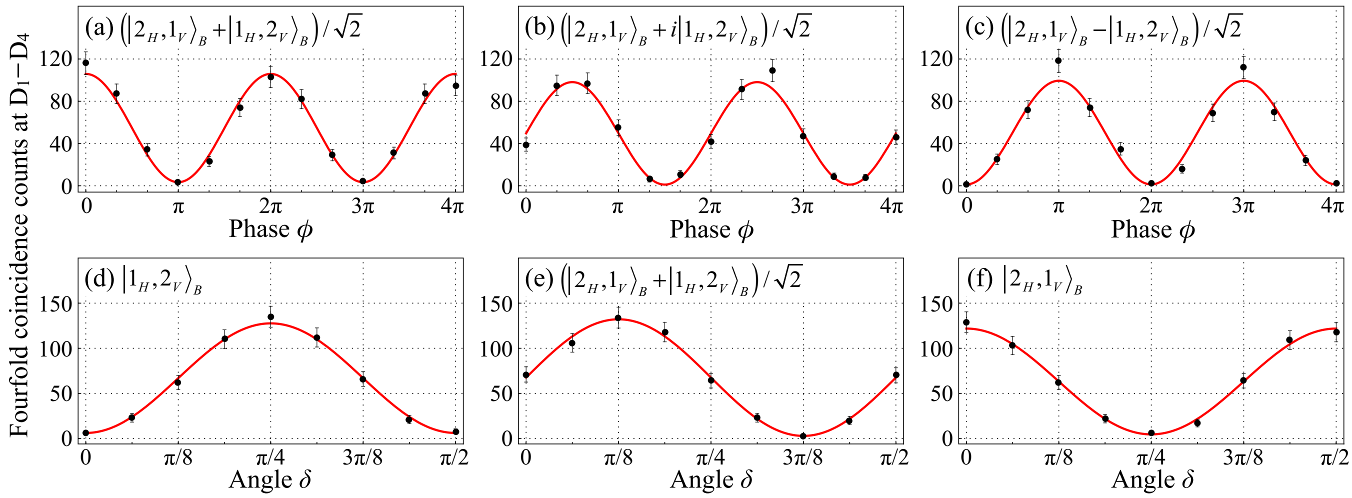

FIG. 2. Measurement of remotely prepared three-photon states. (a)–(c) Remotely prepared three-photon entangled states having different phases, in Eq. (6), are measured by projection onto $\frac{1}{\sqrt{2}}(|2_H,1_V\rangle_B + e^{i\phi}|1_H,2_V\rangle_B)$. (d)–(f) Remotely prepared three-photon states having different amplitudes, in Eq. (7), are measured by projection onto $\cos 2\delta|2_H,1_V\rangle_B + \sin 2\delta|1_H,2_V\rangle_B$. Black dots are experimental data, and red solid lines are sinusoidal fittings to the experimental data. Error bars represent one standard deviation by assuming Poissonian counting statistics. Visibilities of the fittings are (a) $93.8 \pm 1.7\%$, (b) $97.8 \pm 3.1\%$, (c) $97.4 \pm 1.6\%$, (d) $90.8 \pm 1.3\%$, (e) $95.7 \pm 1.0\%$, and (f) $92.7 \pm 1.9\%$.

which is useful in quantum metrology with lossy channels [21–23] and in characterizing the kinds of quantum decoherence [24]. To do this, we employ the single-photon measurement at Alice in Fig. 1, where the angle γ of a half-wave plate H_2 is set to $\pi/8$, the phase shift θ of PS_1 is adjusted. When D_1 clicks and the three photons are directed to Bob, the three-photon entangled state in Eq. (6) is prepared at Bob. For measuring the phase θ in the remotely prepared three-photon state, we employ the measurement setup at Bob in Fig. 1, which projects the three-photon state onto $\frac{1}{\sqrt{2}}(|2_H,1_V\rangle_B + e^{i\phi}|1_H,2_V\rangle_B)$; the phase ϕ of PS_2 is varied while the angle δ of H_3 is set to $\pi/8$, and coincidence counts on D_2, D_3, and D_4 are recorded. By this measurement, the projection probability for the state $|\psi\rangle_B^{\text{phase}}$ is $\frac{1}{2}(1+\cos(\phi-\theta))$, where the offset of the sinusoidal probability as a function of ϕ indicates the phase θ in the three-photon state. Figures 2(a)–2(c) show three-photon entangled states having various phases by choosing different θ values at Alice: For $\theta = 0$, the offset of the sinusoidal oscillation is 0, but as θ is adjusted, the offset is shifted accordingly.

TABLE I. Test of CHSH inequality between the single photon and the three photons. Normalized correlation values $\langle\hat{\lambda}^s\hat{\lambda}^t\rangle$ and the CHSH parameter S_{CHSH} are experimentally obtained. The measurement basis for the single photon is $\{|1_H,0_V\rangle,|0_H,1_V\rangle\}$ and that for the three photons is $\{|2_H,1_V\rangle,|1_H,2_V\rangle\}$. Thus, the Hilbert space dimension for the state under the test is $2\otimes 2$. The experimental S_{CHSH} value violates the classical limit of 2 by more than 7 standard deviations, a clear indication that the single photon and the three photons are entangled. Errors represent one standard deviation, estimated from Poissonian statistics of obtained coincidence counts.

$\langle\hat{\mu}^s\hat{\mu}^t\rangle$	$\langle\hat{\mu}^s\hat{\pi}^t\rangle$	$\langle\hat{\pi}^s\hat{\mu}^t\rangle$	$\langle\hat{\pi}^s\hat{\pi}^t\rangle$	S_{CHSH}
−0.69(5)	0.67(5)	0.64(5)	0.71(4)	2.71(9)

We next carry out RSP of a three-photon state at Bob having a varying degree of entanglement, i.e.,

$$|\psi\rangle_B^{\text{amp}} = \sin 2\gamma|2_H,1_V\rangle_B + \cos 2\gamma|1_H,2_V\rangle_B. \tag{7}$$

To carry out this RSP, the measurement setup at Alice is modified: PS_1 is removed, the angle γ of H_2 is adjusted, and a click at D_1 is informed to Bob. At Bob's side, as it is necessary to observe the change of amplitudes in a three-photon state, the projection basis $\cos 2\delta|2_H,1_V\rangle_B + \sin 2\delta|1_H,2_V\rangle_B$ is used. To do so, PS_2 is removed and coincidence counts at D_2, D_3, and D_4 are measured as a function of the angle δ of H_3. The projection probability of the state $|\psi\rangle_B^{\text{amp}}$ by this measurement is $\frac{1}{2}(1-\cos 4(\delta+\gamma))$, and thus, the amplitude in the three-photon state can be obtained by finding the maximum probability, which takes place at $\delta = \pi/4-\gamma$ (modulo $\pi/2$). Experimental results in Figs. 2(d)–2(f) show the changes of the amplitudes in the three-photon state depending on the single-photon measurement at Alice. Additionally, we have confirmed that contributions of $|3_H,0_V\rangle_B$ and $|0_H,3_V\rangle_B$ in the generated states are negligible, as shown in Fig. 3.

It is important to emphasize that the set of three-photon states in Eq. (7) cannot be fully prepared by linear optical transformation of any particular three-photon states. For example, any nontrivial linear optical transformations on $|2_H,1_V\rangle_B$ necessarily generate undesired quantum states $|3_H,0_V\rangle_B$ and/or $|0_H,3_V\rangle_B$, so that the state $1/\sqrt{2}(|2_H,1_V\rangle_B + |1_H,2_V\rangle_B)$ cannot be prepared [5,25,26]. On the other hand, as RSP is based on measurement-induced nonlinearity [4,5,20], it enables us to access multiphoton states on different orbits determined by linear optical transformations [25]. Our RSP protocol, thus, extends the capability of multiphoton state engineering beyond the linear optical limit via the nonlinearity induced by the single-photon measurement [4,20], allowing us to prepare various multiphoton states required for quantum metrology [21–24] and for fundamental studies in quantum optics [25,26].

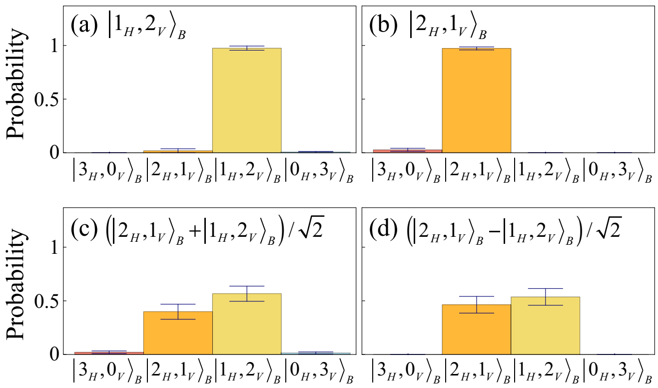

FIG. 3. Contributions of $|3_H,0_V\rangle_B$, $|2_H,1_V\rangle_B$, $|1_H,2_V\rangle_B$, and $|0_H,3_V\rangle_B$ in experimentally generated three-photon states. The maximum contribution from $|3_H,0_V\rangle_B$ and $|0_H,3_V\rangle_B$ is below 0.03. This result also justifies the choice the Hilbert space dimension $2\otimes 2$ as contributions from $|3_H,0_V\rangle_B$ and $|0_H,3_V\rangle_B$ are negligible. Error bars represent one standard deviation, estimated from Poissonian statistics of obtained coincidence counts.

V. GENERALIZATION

We have so far experimentally demonstrated RSP of pure three-photon states, but the scheme can be further generalized to prepare general multiphoton entangled states of arbitrary purity and photon number via single-photon measurement. To prepare a mixed three-photon state at Bob, Alice uses a partial polarizer PP [14] instead of PBS_2 in Fig. 1. This then implements a partial projection on the single photon at Alice of the form,

$$\mathcal{P}(|\phi\rangle_A\langle\phi|,p) = p|\phi\rangle_A\langle\phi| + (1-p)I_A/2, \tag{8}$$

where p is the strength of the projection and I_A is the identity density operator $|1_H,0_V\rangle_A\langle 1_H,0_V| + |0_H,1_V\rangle_A\langle 0_H,1_V|$. Consequently, the three-photon state at Bob becomes a mixed state of the following form,

$$\rho_B = \frac{\mathrm{Tr}_A(\mathcal{P}(|\phi\rangle_A\langle\phi|,p)|\Phi\rangle_{AB}\langle\Phi|)}{\mathrm{Tr}_{AB}(\mathcal{P}(|\phi\rangle_A\langle\phi|,p)|\Phi\rangle_{AB}\langle\Phi|)} = p|\psi\rangle_B\langle\psi| + (1-p)I_B/2, \tag{9}$$

where $I_B = |2_H,1_V\rangle_B\langle 2_H,1_V| + |1_H,2_V\rangle_B\langle 1_H,2_V|$. In other words, an arbitrary amount of noise $(1-p)$ can be added to any three-photon pure state $|\psi\rangle_B$ via Alice's partial projection measurement. Note that a pure three-photon state cannot be transformed to such a mixed state by applying the distinguishability-based decoherence schemes [24,29], which is commonly adopted to generate a mixed single-photon state. This is due to the fact that the distinguishability-based decoherence schemes cause emergence of nontrivial multiphoton state components [24,30,31]; e.g., introducing $|2_H,\tilde{1}_V\rangle_B$, $|1_H,1_V,\tilde{1}_V\rangle_B$, and $|1_H,\tilde{2}_V\rangle_B$, where $|\tilde{n_V}\rangle$ represents the vertically polarized n-photon state in a physically distinguishable mode (i.e., time, path, etc.) from that of $|n_V\rangle$.

Now, for preparing a multiphoton state with arbitrary photon number, we consider n-pair photon generation in the spontaneous parametric down-conversion. The quantum state at the output of PBS_1 in Fig. 1 is then given as $|n_H,n_V\rangle$. If we then consider, after BS_1, the case in which a single photon is reflected to Alice and $2n$-1 photons are directed to Bob, the quantum state shared by Alice and Bob is an entangled state of the form,

$$|\Phi\rangle_{AB}^{(2n)} = 1/\sqrt{2}, \\ (|1_H,0_V\rangle_A|n-1_H,n_V\rangle_B + |0_H,1_V\rangle_A|n_H,n-1_V\rangle_B). \tag{10}$$

For optimizing the generation rate of this state, T:R of BS_1 can be chosen as $2n-1$:1. To carry out RSP at Bob, Alice measures the single photon in the projection basis $|\phi\rangle_A$. This will then project the quantum state of $(2n-1)$ photons at Bob to become,

$$|\psi\rangle_B^{(2n-1)} = \alpha|n_H,(n-1)_V\rangle_B + \beta e^{i\theta}|(n-1)_H,n_V\rangle_B, \tag{11}$$

where the amplitudes α and β as well as the phase θ are determined by the single-photon measurement at Alice. To prepare a mixed state, similarly to above, it is necessary to introduce partial projection measurement $\mathcal{P}(|\phi\rangle_A\langle\phi|,p)$ in Eq. (8). This then results in the multiphoton state of the form,

$$\rho_B^{(2n-1)} = p|\psi\rangle_B^{(2n-1)}\langle\psi| + (1-p)I_B^{(2n-1)}/2, \tag{12}$$

where

$$I_B^{(2n-1)} = |n_H,(n-1)_V\rangle_B\langle n_H,(n-1)_V| \\ + |(n-1)_H,n_V\rangle_B\langle(n-1)_H,n_V|. \tag{13}$$

The RSP by single-photon measurement can, therefore, be applied to multiphoton states, which also provides enhanced engineering capability on multiphoton states.

We finally discuss extension of the RSP scheme via single-photon measurement for preparing an entangled state among three parties: Bob (B), Charlie (C), and David (D). If one employs a three-port beam splitter (implementable by concatenating two two-port beam splitters [19]) to split the three photons in Eq. (2) into three different spatial modes, the three parties located at each output mode share a three-partite entangled state,

$$|\psi\rangle_{\mathrm{BCD}} = \alpha|W\rangle_{\mathrm{BCD}} + \beta e^{i\theta}|\overline{W}\rangle_{\mathrm{BCD}}, \tag{14}$$

where

$$|W\rangle_{\mathrm{BCD}} = (|1_H,0_V\rangle_B|1_H,0_V\rangle_C|0_H,1_V\rangle_D + |1_H,0_V\rangle_B|0_H,1_V\rangle_C|1_H,0_V\rangle_D + |0_H,1_V\rangle_B|1_H,0_V\rangle_C|1_H,0_V\rangle_D)/\sqrt{3},$$
$$|\overline{W}\rangle_{\mathrm{BCD}} = (|1_H,0_V\rangle_B|0_H,1_V\rangle_C|0_H,1_V\rangle_D + |0_H,1_V\rangle_B|1_H,0_V\rangle_C|0_H,1_V\rangle_D + |0_H,1_V\rangle_B|0_H,1_V\rangle_C|1_H,0_V\rangle_D)/\sqrt{3}.$$

Note that $|W\rangle_{\mathrm{BCD}}$ and $|\overline{W}\rangle_{\mathrm{BCD}}$ are genuinely three-partite entangled states [19], and $(|W\rangle_{\mathrm{BCD}} + |\overline{W}\rangle_{\mathrm{BCD}})/\sqrt{2}$ can be used for quantum secret sharing [32]. Therefore, single-photon measurement can also be adopted to remotely prepare a three-partite entangled state by determining amplitudes and phase [α, β, and $e^{i\theta}$ in Eq. (14)] of the entangled state.

VI. CONCLUSIONS

We report an experimental demonstration of remote preparation of three-photon entangled states by measuring only a single photon entangled with the three photons. We have also generalized the RSP protocol to prepare a multiphoton entangled state with arbitrary photon number and purity and to prepare a genuinely three-partite entangled state, both via single-photon measurement. In addition to fundamental interest, our RSP protocol extends the capability of multiphoton state engineering beyond the linear optical limit via the nonlinearity induced by the single-photon measurement [4,20], allowing us to prepare various multiphoton states required for quantum metrology and for fundamental studies in quantum optics: e.g., quantum metrology with lossy channels [21–23], characterization of quantum decoherence [24], and study of quantum polarization [25,26]. We further anticipate that our work will stimulate fundamental studies on entanglement between a single particle and multiple particles, such as, nonlocality tests between a single-particle state and a multiparticle state, quantum teleportation of a single-particle state to a multiparticle state, etc.

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