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Periodic revival of frustrated two-photon creation via interference

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Abstract: It has been known that suitably placed external mirrors can enhance and suppress emission of entangled photon pairs in spontaneous parametric down-conversion (SPDC), known as frustrated two-photon creation via interference. In this work, we report periodic revival of frustrated two-photon creation via interference with SPDC pumped by a continuous-wave (cw) multi-mode laser. As the mirrors are translated relative to the position of the SPDC source, the effect of frustrated two-photon creation via interference gradually dies off. However, as the mirrors are translated even further, the effect of frustrated two-photon creation via interference re-appears periodically. Our theoretical and numerical analyses show that this revival phenomenon is due to the nature of cw multi-mode pump laser. This work clearly demonstrates how the properties of the pump laser, in addition to suitably placed external mirrors, can be used to modify the process of spontaneous two-photon emission.

1. Introduction

Quantum interference, i.e., interference of a quantum mechanical object, is one of the cornerstones of quantum physics and is at the very heart of the so-called the second quantum revolution, which aims to take advantage of quantum superposition and quantum entanglement for a variety of applications [1]. Such potential applications include quantum communication [2,3], quantum teleportation [4–7], quantum cryptography [8], quantum imaging [9–12], quantum metrology [13], and quantum computing [14,15]. Proof-of-principle demonstrations of these potential applications have been carried out with quantum states of light and the two-photon state of spontaneous parametric down-conversion (SPDC) has been the workhorse in the field of experimental quantum optics [16].

Starting from the Shih-Alley/Hong-Ou-Mandel two-photon interference [17,18], SPDC has been used extensively to study the second-order quantum interference in degrees of freedom, such as, polarization [19,20], energy-time [21–23], time-bin [24–26] and position-momentum [9–11,27]. Moreover, many unique features of multi-partite quantum interference and entanglement have been studied and discovered by using the two-photon states of SPDC [28–34]. SPDC photons have also been used to probe a number of unique situations in which first-order interference amounts to quantum optical interference [35–38]. An interesting feature of such first-order quantum optical interference is that the interference of the 'signal' photon appears to have been affected by the phases of the pump laser and the undetected 'idler' photon.

Similarly to the Purcell effect [39], the spontaneous emission rate of the photon pair in the SPDC process can be enhanced or suppressed by suitably placed external mirrors, leading to the effect of frustrated two-photon creation [40]. As the effect is of the first-order in the SPDC intensity, the coherence length of the SPDC photons as well as that of the pump laser limit the spatial ranges over which frustrated two-photon creation via interference may be observed [41,42].

In this work, we report periodic revival of frustrated two-photon creation via interference with SPDC pumped by a continuous-wave (cw) multi-mode laser. As the external mirrors are translated relative to the position of the SPDC source beyond the coherence length, the effect of frustrated two-photon creation via interference gradually dies off as shown in [40]. However, as
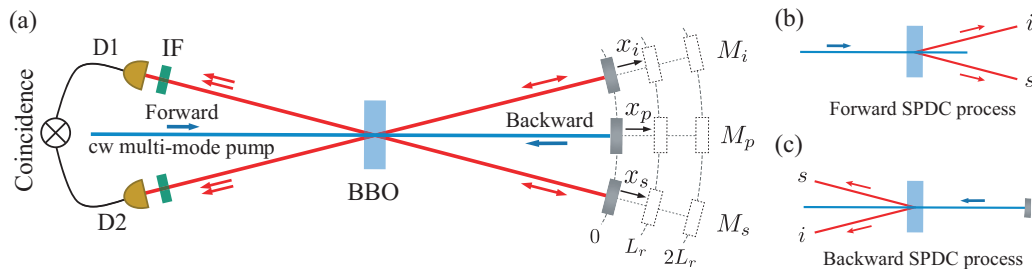
Fig. 1. (a) Schematic of the experimental setup. The forward and backward SPDC generation processes are shown in (b) and (c), respectively. The initial positions of the mirrors, all at the same distances from the BBO crystal, are identified with 0. The mirrors scanned around the positions 0, L_r, and $2L_r$, where L_r is the coherence revival period of the pump laser. The idler (M_i), the pump (M_p), and the signal mirror (M_s) positions are denoted, respectively, as x_i, x_p and x_s. IF is an interference filter.

the mirrors are translated even further, the effect of frustrated two-photon creation via interference reappears periodically. Our theoretical and numerical analyses show that this revival phenomenon is due to the nature of cw multi-mode pump laser, providing an effective periodic boundary condition for the SPDC process [26, 43–45]. This work clearly demonstrates how the properties of the pump laser and suitably placed external mirrors can be used to modify the process of spontaneous two-photon emission via multi-mode quantum interference.

2. Experiment

The experimental setup is schematically shown in Fig. 1. A 2 mm-thick type-I β-barium borate (BBO) crystal is pumped by a cw multi-mode diode laser operating at 405 nm with the full width at half maximum (FWHM) bandwidth of 1 nm. The coherence revival period of the pump laser is measured to be $L_r = 2.895$ mm ($\pm$ 0.008 mm) which is measured by using an unbalanced Michelson interferometer [43]. The 810 nm degenerate photon pair is generated in the non-collinear geometry at an angle of 3° between the pump and the SPDC photons. As the pump laser is reflected back by the pump mirror (M_p), there are two possible ways of 810 nm photon pair generation via the SPDC process; the photon pair may be generated by the forward pump (see Fig. 1(b)) or by the backward pump (see Fig. 1(c)). By suitably placing the external idler mirror (M_i) and the signal mirror (M_s), the spatial mode of the forward SPDC process overlaps with that of the backward SPDC process. The forward and backward biphoton probability amplitudes may then exhibit quantum interference, resulting in phase-dependent suppression or enhancement of spontaneous two-photon creation. The photons are collected into single mode fibers and then detected at the Si avalanche single photon detectors (D1 and D2) after passing through the 10-nm FWHM bandwidth interference filters. The single and the coincidence count rates of the two detectors are monitored while the external mirrors are translated by the motorized stages.

Periodic revival of frustrated two-photon creation via interference is observed by scanning externally placed mirrors M_p, M_s, and M_i. First, the signal mirror M_s is scanned while the pump mirror M_p and the idler mirror M_i are fixed at $x_i = x_p = 0$ in Fig. 1. As shown in Fig. 2, the coincidence and single count rates exhibit sinusoidal modulation as a function of the signal mirror position. Interestingly, even though only the signal mirror is scanned, the idler detector D2 also exhibits interference, indicating that the observed first- and second-order interference effects are due to genuine quantum interference. As the signal mirror position goes outside the coherence length of the signal photon defined by the 10 nm FWHM spectral filter, the interference effects fade away gradually. Since the periodic revival feature of frustrated two-photon creation does not appear in this case, the scan range of the data in Fig. 2 only covers the coherence
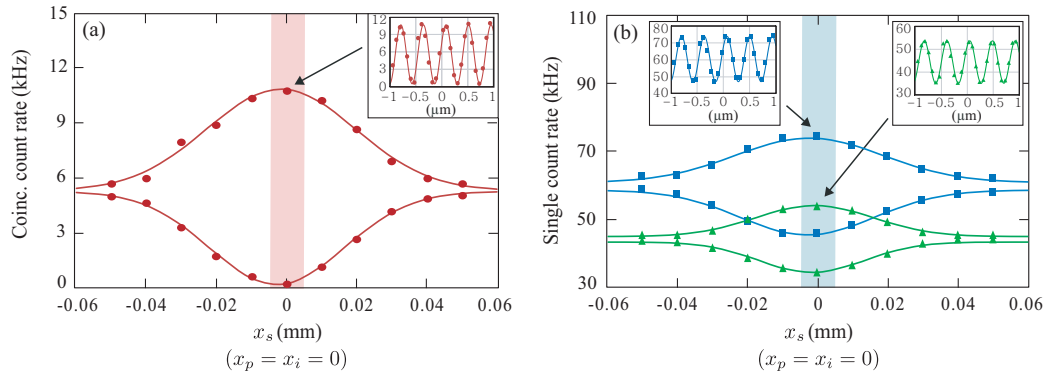
Fig. 2. The signal mirror M_s position x_s is scanned while the pump mirror M_p and the idler mirror M_i are fixed at $x_i = x_p = 0$. The coincidence count (a) and the single count (b) rates exhibit sinusoidal modulation as shown in the inset. The interference envelope is shown by displaying the maximum and minimum points of the data around a series of mirror positions. The maximum visibility of the coincidence count rate is 96.5%. The solid circle, the square, and the triangle data points refer to the coincidence, the signal, and the idler count rates, respectively. The solid lines refer to fit to the data (a Gaussian for the envelope and a sinusoidal fit for the signal and idler).

length of the signal photon. The effect of periodic revival of frustrated two-photon creation via interference will be demonstrated scanning the pump mirror M_p. Note that the first- and second-order interference in Fig. 2 show sinusoidal fringes having the period of 810 nm.

To observe periodic revival of frustrated two-photon creation via interference, the pump mirror M_p is scanned while the signal and idler mirrors M_s and M_i are fixed at $x_s = x_i = 0$. As shown in Fig. 3, the coincidence and the single count rates of the detectors exhibit the familiar second- and first-order interference, respectively, when the pump mirror M_p is scanned within the coherence length. In Fig. 3(a), the conditions of the external mirrors are identical to that of Fig. 2 and the visibility of the interference dies off gradually as the relative difference between the position of the pump mirror and the positions of the signal/idler mirrors approaches the coherence length of the pump laser. However, when the pump mirror M_p is scanned even more to approach the pump coherence revival length $L_r = 2.895$ mm, we find that the second- and the first-order interference re-emerges, see Fig. 3(b). As before, the envelopes of the interference fringes are determined by the coherence length of the pump laser. Finally, in Fig. 3(c), the pump mirror M_p is translated even farther to $2L_r$ while keeping the signal and idler mirrors M_s and M_i at $x_s = x_i = 0$. Clearly, the similar interference is observed and slight reduction of visibility is caused by the alignment issue when the path length difference becomes large (i.e., it becomes more difficult to perfectly overlap the spatial modes of the forward and the backward SPDC as the path length difference becomes large). We have thus observed emergence of the first- and the second-order interference at the period of L_r, periodic revival of frustrated two-photon creation via interference. The interference fringes shown in the inset of Fig. 3 indicate that the single and the coincidence count rates have sinusoidal modulation at the period of pump laser wavelength, i.e., the modulation is twice as fast as that of Fig. 2 in which the signal mirror M_s is scanned. The data in Fig. 3 clearly show that the revival property of the pump is transferred to frustrated two-photon creation via interference [26, 43–45].

The effect of periodic revival of frustrated two-photon creation via interference is also observed by scanning the signal and idler mirrors M_s and M_i simultaneously, while keeping the pump mirror M_p fixed at $x_p = 0$. As shown in Fig. 4(a), the coincidence and the single count rates of the detectors exhibit the second- and first-order interference, respectively, when the mirrors
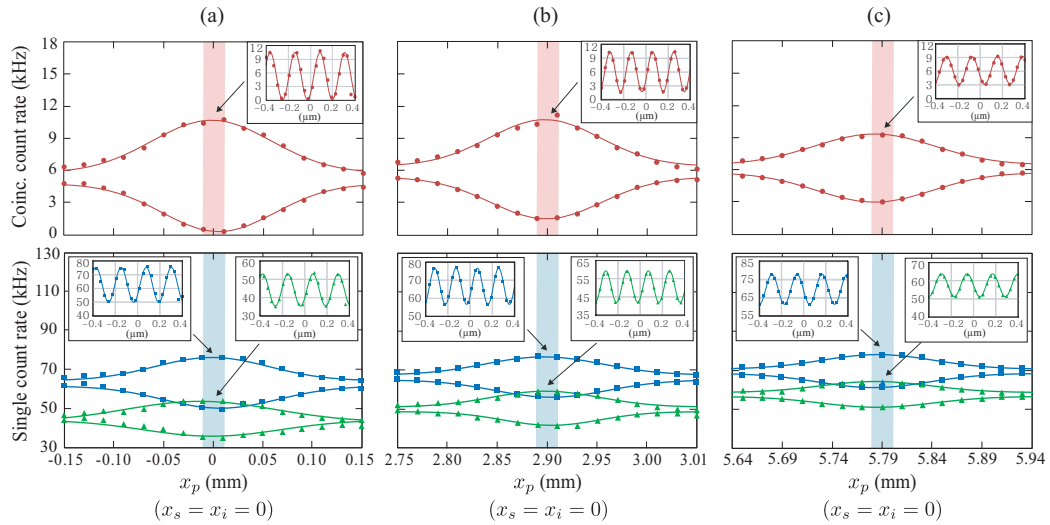
Fig. 3. The pump mirror M_p position x_p is scanned while the signal mirror M_s and the idler mirror M_i are fixed at $x_s = x_i = 0$. The coarse positions of the pump mirror M_p are set at (a) $x_p = 0$, (b) $x_p = L_r$, and (c) $x_p = 2L_r$. Experimentally, the pump coherence revival length L_r is determined to be 2.895 mm. The interference envelope is shown by displaying the maximum and minimum points of the data around a series of mirror positions. The insets show the interference fringe at the center of each envelope. The maximum visibilities for the coincidence count rates are (a) 94.2%, (b) 73.4% and (c) 50.5%. The solid circle, the square, and the triangle data points refer to the coincidence, the signal, and the idler count rates, respectively. The solid lines refer to fit to the data (a Gaussian for the envelope and a sinusoidal fit for the signal and idler).

M_s and M_i are simultaneously scanned within the SPDC coherence length. The visibility of the interference dies off gradually as the relative difference between the position of the pump mirror and the positions of the signal/idler mirrors approaches the coherence length of the pump laser. However, when the mirrors M_s and M_i are scanned even more to approach the pump coherence revival length $L_r = 2.895$ mm, we find that the second- and the first-order interference re-emerges, see Fig. 4(b). Finally, in Fig. 4(c), the mirrors M_s and M_i are translated even farther to $2L_r$ while keeping the pump mirror at $x_p = 0$. Clearly, the similar interference is observed and, as before, slight reduction of visibility is caused by the alignment issue when the path length difference becomes large. The data in Fig. 4 are similar to the data in Fig. 3 in that the interference fringes have the period of the pump laser wavelength., i.e., the modulation is twice as fast as that of Fig. 2 in which only the signal mirror M_s is scanned. The experimental data therefore demonstrate clearly that spontaneous two-photon emission can be suppressed or enhanced with an effective periodic boundary condition derived from the coherence revival property of the cw multi-mode pump laser.

3. Theory

The two-photon state ρ of spontaneous parametric down-conversion pumped by a multi-mode laser may be written as [26, 44]

$$\rho = \int d\omega_p S(\omega_p) |\psi(\omega_p)\rangle \langle \psi(\omega_p)|. \tag{1}$$
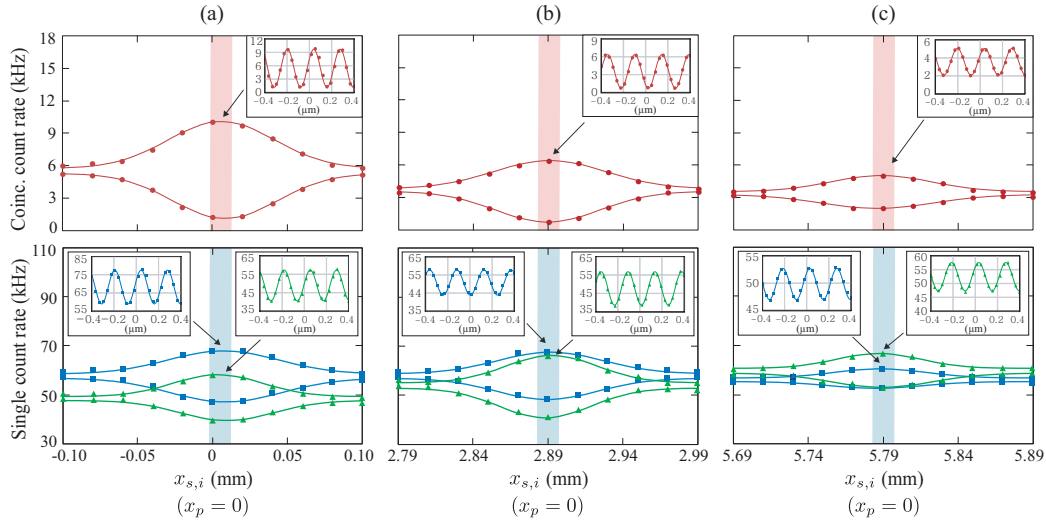
Fig. 4. The signal mirror M_s and the idler mirror M_i are scanned simultaneously while the pump mirror M_p position x_p is fixed at $x_p = 0$. The coarse positions of the signal/idler mirrors M_s and M_i are set at (a) $x_{s,i} = 0$, (b) $x_{s,i} = L_r$, and (c) $x_{s,i} = 2L_r$. Experimentally, the pump coherence revival length L_r is determined to be 2.89 mm. The interference envelope is shown by displaying the maximum and minimum points of the data around a series of mirror positions. The insets show the interference fringe at the center of each envelope. The maximum visibilities for the coincidence count rates are (a) 80.1%, (b) 80.3% and (c) 43.2%. The solid circle, the square, and the triangle data points refer to the coincidence, the signal, and the idler count rates, respectively. The solid lines refer to fit to the data (a Gaussian for the envelope and a sinusoidal fit for the signal and idler).

Here, $S(\omega_p)$ is the spectral power density of the pump laser given by the incoherent sum of discrete frequency modes,

$$S(\omega_p) = \frac{\sum_{n=-N}^{N} S_0(\omega_p)\delta(\omega_p - \omega_{p0} - n\mu)}{\sum_{n=-N}^{N} S_0(\omega_{p0} + n\mu)}, \qquad (2)$$

where ω_{p0}, μ, and n are the central frequency of the pump, the mode spacing, and the mode number, respectively. We assume that the overall spectral profile is Gaussian, $S_0(\omega_p) = \exp[-(\omega_p - \omega_{po})^2/2\sigma_p^2]$ with the pump bandwidth σ_p. The two-photon quantum state of SPDC, pumped by a single-mode laser with frequency ω_p, at the detectors D1 and D2 is $|\psi(\omega_p)\rangle$ and, in our experimental setup, it is described by quantum superposition of the forward SPDC and the backward SPDC processes. Therefore, it is calculated to be

$$|\psi(\omega_p)\rangle = \int d\omega_s d\omega_i \delta(\omega_p - \omega_s - \omega_i)\text{sinc}(\Delta_k l/2)e^{i\Delta_k l/2}\mathcal{F}_s(\omega_s)\mathcal{F}_i(\omega_i)(e^{i\phi_f} + e^{i\phi_b})|\omega_s, \omega_i\rangle. \quad (3)$$

The subscripts p, s and i stand for the pump, the signal, and the idler, respectively. The thickness of the SPDC crystal is l and the phase mismatch is $\Delta_k = k_p - k_s - k_i$. As the forward SPDC process is affected by the translation of M_s and M_i, the dynamical phase of the two-photon state of the forward SPDC process ϕ_f is given by $\phi_f = \frac{2}{c}(\omega_s x_s + \omega_i x_i)$. The dynamical phase of the two-photon state due to the backward SPDC process ϕ_b is adjusted by scanning M_p and, therefore, is given by $\phi_b = \frac{2}{c}\omega_p x_p$. Additionally, for the purpose of numerical simulation, the spectral filters introduced in front of the detectors are assumed to have the Gaussian transmission function, $\mathcal{F}_{s,i}(\omega) = \exp[-(\omega - \omega_{0s,0i})^2)/2\sigma_\mathcal{F}^2]/\sqrt{\sigma_\mathcal{F}\sqrt{\pi}}$, where $\omega_{0s,0i}$ and $\sigma_\mathcal{F}$ are the central
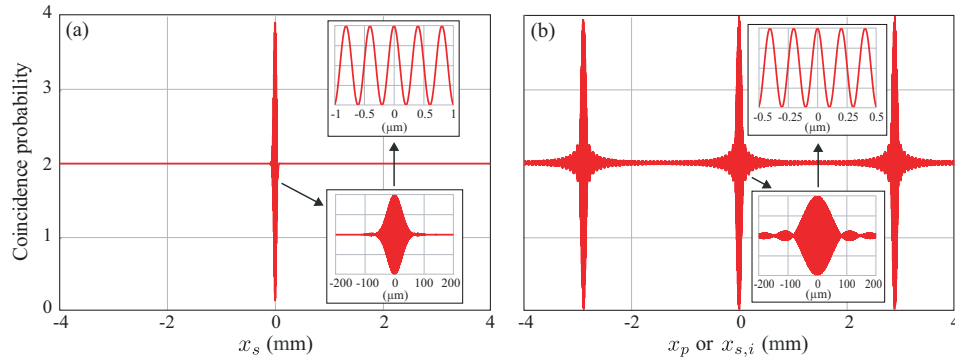
Fig. 5. Results of the theoretical calculations. (a) Coincidence count rates when only the signal mirror M_s is scanned. Periodic revival of frustrated two-photon creation via interference does not occur. (b) Coincidence count rates when the M_p is translated or when both the M_s and M_i are translated simultaneously. Periodic revival of frustrated two-photon creation via interference appears in this case and the numerical simulation matches well with the experimental data.

frequency of the signal, idler photon and bandwidth of the filter, respectively. Note that, for type-I SPDC, the SPDC spectrum produced by the $\text{sinc}(\Delta_k l/2)$ term is much broader than the width of the interference filters used in the experiment. Thus, the $\text{sinc}(\Delta_k l/2)$ term may be approximated to unity for the purpose of numerical simulation.

The coincidence count rate R between the two detectors D1 and D2 is given by

$$R \propto \int_{-\infty}^{\infty} dt_1 dt_2 \text{Tr}[\rho E_1^{(-)}(t_1) E_2^{(-)}(t_2) E_2^{(+)}(t_2) E_1^{(+)}(t_1)], \tag{4}$$

where $E_j^{(+)}(t) = \int_{-\infty}^{\infty} d\omega a_j(\omega) e^{-i\omega t}$ and $a_j(\omega)$ is an annihilation operator for a photon of frequency ω in the j mode. The detection times at the detectors D1 and D2 are, respectively, t_1 and t_2. Evaluating the Eq. (4) by substituting Eq. (1) for ρ leads to the coincidence count rate,

$$R = \sum_{\omega_p=\omega_{p0}-N\mu}^{\omega_{p0}+N\mu} \int d\omega_s |\mathcal{F}_s(\omega_s)|^2 |\mathcal{F}_i(\omega_p - \omega_s)|^2 \left(2 + 2\cos\left[\frac{2}{c}(\omega_p x_p - \omega_s x_s - (\omega_p - \omega_s)x_i)\right]\right). \tag{5}$$

For evaluating the above result numerically, all parameters are set to the experimental conditions. For the conditions of the pump laser, the number of modes is $N = 20$, the central wavelength is 405 nm, and the FWHM bandwidth is 1 nm. The mode spacing $\mu = 0.0567$ nm is determined from a separate coherence revival experiment with an unbalanced Michelson interferometer. The mode spacing μ determines the pump coherence revival period L_r with the relation $L_r = 2\pi c/\mu = 2.895$ mm [43]. The FWHM bandwidth of the filter transmission function is set to be 10 nm at 810 nm center wavelength.

Figure 5 shows the numerical simulation result of Eq. (5). Let us first consider the case where M_s is scanned. Figure 5(a) shows the coincidence count rates as a function of the x_s. It is clearly seen that scanning M_s or M_i alone would not produce the periodic revival of frustrated two-photon creation via interference. This is due to the fact that, as scanning of M_s only affects the signal mode, the forward and backward SPDC probability amplitudes do not interfere when the M_s is translated over the coherence length, even though the pump has an effective periodic boundary condition. Periodic revival of frustrated two-photon creation via interference, however, is observed when the pump mirror M_p is scanned, as shown in Fig. 5(b). The numerical results

are in good agreement with the experimental observations, in particular, the periodic nature of frustrated two-photon creation via interference due to the multimode pump is well explained.

4. Conclusion

In the classic frustrated two-photon creation via interference experiment [40], it has been known that suitably placed external mirrors can enhance and suppress emission of entangled photon pairs in spontaneous parametric down-conversion (SPDC). As the effect is of the first-order in the SPDC intensity, the coherence length of the SPDC photons as well as that of the pump laser limit the spatial ranges over which frustrated two-photon creation via interference may be observed. In this work, we report the experimental and theoretical studies on the effect of cw multi-mode laser on frustrated two-photon creation via interference. As the mirrors are translated relative to the position of the SPDC source, the effect of frustrated two-photon creation via interference gradually dies off. However, as the mirrors are translated even further, the effect of frustrated two-photon creation via interference re-appears periodically. And, this period is found to be identical to the interference revival period of the cw multi-mode diode laser [43]. Our theoretical analysis shows that, indeed, the cw multi-mode pump laser provides an effective periodic boundary condition for the SPDC process. This work clearly demonstrates how the properties of the pump laser, in addition to suitably placed external mirrors, can be used to modify the process of spontaneous two-photon emission via multi-mode quantum interference.

References

1. J. P. Dowling, and G. J. Milburn, "Quantum technology: the second quantum revolution," Philos. Trans. Royal Soc. A **361**(1809), 1655–1674 (2003).
2. N. Gisin, and R. Thew, "Quantum communication," Nat. Photonics **1**(3), 165–171 (2007).
3. R. Ursin, F. Tiefenbacher, T. Schmitt-Manderbach, H. Weier, T. Scheidl, M. Lindenthal, B. Blauensteiner, T. Jennewein, J. Perdigues, P. Trojek, B. Ömer, M. Fürst, M. Meyenburg, J. Rarity, Z. Sodnik, C. Barbieri, H. Weinfurter, and A. Zeilinger, "Entanglement-based quantum communication over 144 km," Nat. Phys. **3**(7), 481–486 (2007).
4. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels," Phys. Rev. Lett. **70**(13), 1895–1899 (1993).
5. D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," Nature **390**(6660), 575–579 (1997).
6. Y.-H. Kim, S. P. Kulik, and Y. Shih, "Quantum teleportation of a polarization state with a complete Bell state measurement," Phys. Rev. Lett. **86**(7), 1370–1373 (2001).
7. X.-L. Wang, X.-D. Cai, Z.-E. Su, M.-C. Chen, D. Wu, L. Li, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, "Quantum teleportation of multiple degrees of freedom of a single photon," Nature **518**(7540), 516–519 (2015).
8. N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, "Quantum cryptography,"' Rev. Mod. Phys. **74**(1), 145 (2002).
9. T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, "Optical imaging by means of two-photon quantum entanglement," Phys. Rev. A **52**(5), R3429–R3432 (1995).
10. D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, "Observation of two-photon "ghost" interference and diffraction," Phys. Rev. Lett. **74**(18), 3600–3603 (1995).
11. M. D'Angelo, Y.-H. Kim, S. P. Kulik, and Y. Shih, "Identifying entanglement using quantum ghost interference and imaging," Phys. Rev. Lett. **92**(23), 233601 (2004).
12. G. Brida, M. Genovese and I. R. Berchera, "Experimental realization of sub-shot-noise quantum imaging," Nat. Photonics **4**(4), 227–230 (2010).
13. V. Giovannetti, S. Lloyd, and L. Maccone, "Advances in quantum metrology," Nat. Photonics **5**(4), 222–229 (2011).

14. P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, "Linear optical quantum computing with photonic qubits," Rev. Mod. Phys. **79**(1), 135–174 (2007).

15. X.-C. Yao, T.-X. Wang, H.-Z. Chen, W.-B. Gao, A. G. Fowler, R. Raussendorf, Z.-B. Chen, N.-L. Liu, C.-Y. Lu, Y.-J. Deng, Y.-A. Chen, and J.-W. Pan, "Experimental demonstration of topological error correction," Nature **482**(7386), 489–494 (2012).

16. D. N. Klyshko, *Photons and Nonlinear Optics* (Gordon and Breach Science, 1988).

17. C. O. Alley and Y. Shih, in *Proceedings of the Second International Symposium on Foundations of Quantum Mechanics in the Light of New Technology* (Physical Society of Japan, 1986), p. 47.

18. C. K. Hong, Z. Y. Ou, and L. Mandel, "Measurement of subpicosecond time intervals between two photons by interference," Phys. Rev. Lett. **59**(18), 2044–2046 (1987).

19. Y. H. Shih and C. O. Alley, "New type of Einstein-Podolsky-Rosen-Bohm experiment using pairs of light quanta produced by optical parametric down conversion," Phys. Rev. Lett. **61**(26), 2921–2924 (1988).

20. Z. Y. Ou and L. Mandel, "Violation of Bell's inequality and classical probability in a two-photon correlation experiment," Phys. Rev. Lett. **61**(1), 50–53 (1988).

21. J. D. Franson, "Bell inequality for position and time," Phys. Rev. Lett. **62**(19), 2205–2208 (1989).

22. Y. H. Shih, A. V. Sergienko, and M. H. Rubin, "Einstein-Podolsky-Rosen state for space-time variables in a two-photon interference experiment," Phys. Rev. A. **47**(2), 1288–1293 (1993).

23. P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, "High-visibility interference in a Bell-inequality experiment for energy and time," Phys. Rev. A. **47**(4), R2472–R2475 (1993).

24. W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, "Quantum cryptography using entangled photons in energy-time Bell states," Phys. Rev. Lett. **84**(20), 4737–4740 (2000).

25. T. Honjo, H. Takesue, and K. Inoue, "Generation of energy-time entangled photon pairs in 1.5-μm band with periodically poled lithium niobate waveguide," Opt. Express **15**(4), 1679–1683 (2007).

26. O. Kwon, K.-K. Park, Y.-S. Ra, Y.-S. Kim, and Y.-H. Kim, "Time-bin entangled photon pairs from spontaneous parametric down-conversion pumped by a cw multi-mode diode laser," Opt. Express **21**(21), 25492–25500 (2013).

27. J.-C. Lee, K.-K. Park, T.-M. Zhao, and Y.-H. Kim, "Einstein-Podolsky-Rosen entanglement of narrow-band photons from cold atoms," Phys. Rev. Lett. **117**(25), 250501 (2016).

28. P. G. Kwiat, A. J. Berglund, J. B. Altepeter, and A. G. White, "Experimental verification of decoherence-free subspaces," Science **290**(5491), 498–501 (2000).

29. J.-W. Pan, S. Gasparoni, R. Ursin, G. Weihs, and A. Zeilinger, "Experimental entanglement purification of arbitrary unknown states," Nature **423**(6938), 417–422 (2003).

30. P. Walther, J.-W. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni, and A. Zeilinger, "De Broglie wavelength of a non-local four-photon state," Nature **429**(6988), 158–161 (2004).

31. M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, "Environment-induced sudden death of entanglement," Science **316**(5824), 579–582 (2007).

32. Y.-S. Kim, J.-C. Lee, O. Kwon, and Y.-H. Kim, "Protecting entanglement from decoherence using weak measurement and quantum measurement reversal," Nature Phys. **8**(2), 117–120 (2012).

33. Y.-S. Ra, M. C. Tichy, H.-T. Lim, O. Kwon, F. Mintert, A. Buchleitner, and Y.-H. Kim, "Nonmonotonic quantum-to-classical transition in multiparticle interference," Proc. Natl. Acad. Sci. U.S.A. **110**(4), 1227–1231 (2013).

34. L.-K. Chen, Z.-D. Li, X.-C. Yao, M. Huang, W. Li, H. Lu, X. Yuan, Y.-B. Zhang, X. Jiang, C.-Z. Peng, L. Li, N.-L. Liu, X. Ma, C.-Y. Lu, Y.-A. Chen, and J.-W. Pan, "Observation of ten-photon entanglement using thin BiB_3O_6 crystals," Optica **4**(1), 77–83 (2017).

35. X. Y. Zou, L. J. Wang, and L. Mandel, "Induced coherence and indistinguishability in optical interference," Phys. Rev. Lett. **67**(3), 318–321 (1991).

36. L. J. Wang, X. Y. Zou, and L. Mandel, "Induced coherence without induced emission," Phys. Rev. A. **44**(7), 4614-4622 (1991).

37. A. V. Burlakov, M. V. Chekhova, D. N. Klyshko, S. P. Kulik, A. N. Penin, Y. H. Shih, and D. V. Strekalov, "Interference effects in spontaneous two-photon parametric scattering from two macroscopic regions," Phys. Rev. A. **56**(4), 3214–3225 (1997).

38. Y.-H. Kim, M. V. Chekhova, S. P. Kulik, Y. Shih, and M. H. Rubin, "First-order interference of nonclassical light emitted spontaneously at different times," Phys. Rev. A. **61**(5), 051803 (2000).

39. E. M. Purcell, "Spontaneous emission probabilities at radio frequencies," Phys. Rev. **69**, 681 (1946).

40. T. J. Herzog, J. G. Rarity, H. Weinfurter, and A. Zeilinger, "Frustrated two-photon creation via interference," Phys. Rev. Lett. **72**(5), 629–632 (1994).

41. P. H. Souto Ribeiro, and G. A. Barbosa, "Mirror effects and induced coherence in parametric down-conversion," Opt. Commun. **139**(1-3), 139–147 (1997).

42. A. K. Jha, M. N. O'Sullivan, K. W. C. Chan, and R. W. Boyd, "Temporal coherence and indistinguishability in two-photon interference effects," Phys. Rev. A **77**(2), 021801 (2008).

43. S.-Y. Baek, O. Kwon, and Y.-H. Kim, "High-resolution mode-spacing measurement of the blue-violet diode laser using interference of fields created with time delays greater than the coherence time," Jpn. J. Appl. Phys. **46**(12R), 7720–7723 (2007).

44. O. Kwon, Y.-S. Ra, and Y.-H. Kim, "Coherence properties of spontaneous parametric down-conversion pumped by a multi-mode cw diode laser," Opt. Express **17**(15), 13059–13069 (2009).

45. A. V. Burlakov, M. V. Chekhova, O. A. Karabutova, and S. P. Kulik, "Biphoton interference with a multimode pump," Phys. Rev. A **63**(5), 053801 (2001).