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Experimental Realization of Popper's Experiment: Violation of the Uncertainty Principle?

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An entangled pair of photons (1 and 2) are emitted in opposite directions. A narrow slit is placed in the path of photon 1 to provide the precise knowledge of its position on the y-axis and this also determines the precise y-position of its twin, photon 2, due to quantum entanglement. Is photon 2 going to experience a greater uncertainty in momentum, that is, a greater Δp_y because of the precise knowledge of its position y? The experimental data show $\Delta y \, \Delta p_y < h$ for photon 2. Can this recent realization of the thought experiment of Karl Popper signal a violation of the uncertainty principle?

1. INTRODUCTION

Uncertainty, one of the basic principles of quantum mechanics, distinguishes the world of quantum phenomena from the realm of classical physics. Quantum mechanically, one can never expect to measure both the precise position and momentum of a particle at the same time. It is prohibited. We say that the quantum observables "position" and "momentum" are "complementary" because the precise knowledge of the position (momentum) implies that all possible outcomes of measuring the momentum (position) are equally probable.

Karl Popper, being a "metaphysical realist", however took a different point of view. In his opinion, the quantum formalism *could* and *should* be interpreted realistically: a particle must have precise position and momentum, which shares the same view as Einstein. In this regard he invented a thought experiment in the early 1930's which aimed to support the realistic interpretation of quantum mechanics and undermine the Copenhagen

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orthodoxy.[1] What Popper intends to show in his thought experiment is that a particle can have both precise position and momentum at the same time through the correlation measurement of an entangled two-particle system. This bears striking similarity to what EPR *gedankenexperiment* of 1935 seeks to conclude.[2] But different from EPR's *gedankenexperiment*, the physics community remained ignorant of Popper's experiment.

In this paper we wish to report a recent realization of Popper's thought experiment. Indeed, it is astonishing to see that the experimental results agree with Popper's prediction. Through quantum entanglement one may learn the precise knowledge of a photon's position and would therefore expect a greater uncertainty in its momentum under the usual Copenhagen interpretation of the uncertainty relations. However, the measurement shows that the momentum does not experience a corresponding increase of uncertainty. Is this a violation of the uncertainty principle?

As a matter of fact, one should not be surprised with the experimental result and should not consider this question as a new challenge. Similar results have been demonstrated in EPR type of experiments and the same question has been asked in EPR's 1935 paper.[2] In the past decades, we have been worrying about problems concerning causality, locality, and reality more than the "crux" of the EPR paradox itself: the uncertainty principle.

2. POPPER'S EXPERIMENT

Similar to the EPR's *gedankenexperiment*, Popper's experiment is also based on the feature of *two-particle entanglement*. Quantum mechanics allows the entangled EPR-type state, a state in which if the position or momentum of particle 1 is known the corresponding observable of its twin, particle 2, is then 100% determined.[2] Popper's original thought experiment is schematically shown in Fig. 1. A point source S, positronium as Popper suggests, is placed at the center of the experimental arrangement from which entangled pairs of particles 1 and 2 are emitted in opposite directions along the respective positive and negative x-axes towards two screens A and B. There are slits on both screens parallel to the y-axis and the slits may be adjusted by varying their widths Δy. Beyond the slits on each side stand an array of Geiger counters for the coincidence measurements of the particle pairs as shown in the figure. The entangled pair could be emitted to any direction in 4π solid angles from the point source. However, if particle 1 is detected in a certain direction then particle 2 is known to be in the opposite direction due to the momentum conservation of the quantum pair.

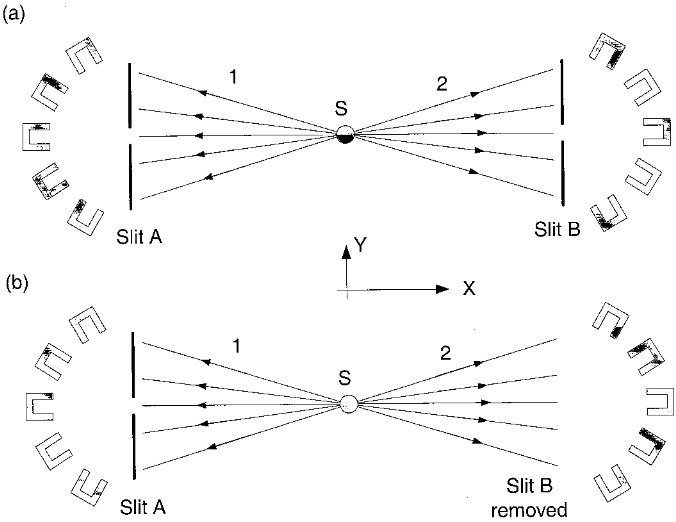

Fig. 1. Popper's proposed experiment. An entangled pair of particles are emitted from a point source with momentum conservation. A narrow slit on screen A is placed in the path of particle 1 to provide the precise knowledge of its position on the y-axis and this also determines the precise y-position of its twin, particle 2 on screen B. (a) Slits A and B are adjusted both very narrowly. (b) Slit A is kept very narrow and slit B is left wide open.

First, let us imagine the case in which slits A and B are adjusted both very narrowly. In this circumstance, counters should come into play which are higher up and lower down as viewed from the slits. The firing of these counters is indicative of the greater Δp_y due to the smaller Δy for each particle. There seems to be no disagreement in this situation between both the Copenhagen school and Popper and both sides can provide a reasonable explanation according to their own philosophical beliefs.

Next, suppose we keep the slit at A very narrow and leave the slit at B wide open. The main purpose of the narrow slit A is to provide the precise knowledge of the position y of particle 1 and this subsequently determines the precise position of its twin (particle 2) on side B through quantum entanglement. Now, asks Popper, in the absence of the physical interaction with an actual slit, does particle 2 experience a greater uncertainty in Δp_y due to the precise knowledge of its position? Based on his "statistical-scatter" theory, Popper provides a straightforward prediction: *particle* 2 *must not experience a greater* Δp_y *unless a real physical narrow slit B is applied*. However, if Popper's conjecture is correct, this would imply the product of Δy and Δp_y of particle 2 could be smaller than

$h(\Delta y\ \Delta p_y < h)$. This may pose a serious difficulty for the Copenhagen camp and perhaps for many of us. On the other hand, if particle 2 going to the right does scatter like its twin which has passed though slit A, even though slit B is wide open, we are then confronted with an apparent *action-at-a distance*!

3. REALIZATION OF POPPER'S EXPERIMENT

We have realized Popper's experiment with the use of the entangled two-photon source of spontaneous parametric down conversion (SPDC).[3,4] In order to clearly demonstrate all aspects of the historical and modern experimental concerns in a practical manner, Popper's original design is slightly modified as shown in Fig. 2. The two-photon source is a CW Argon ion laser pumped SPDC which provides a two-photon entangled state that preserves momentum conservation for the signal-idler photon pair in the SPDC process. By taking advantage of the nature of entanglement of the signal-idler pair (also labeled "photon 1" and "photon 2") one

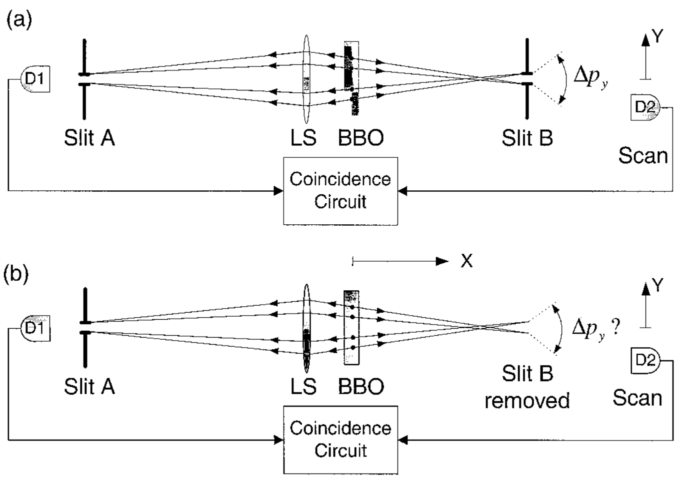

Fig. 2. Modified version of Popper's experiment. An EPR photon pair is generated by SPDC. A lens and a narrow slit A are placed in the path of photon 1 to provide the precise knowledge of its position on the y-axis and also determines the precise y-position of its twin, photon 2, on screen B due to a "ghost image" effect. Two detectors D_1 and D_2 are used to scan in the y-directions for coincidence counts. (a) Slits A and B are adjusted both very narrowly. (b) Slit A is kept very narrow and slit B is left wide open.

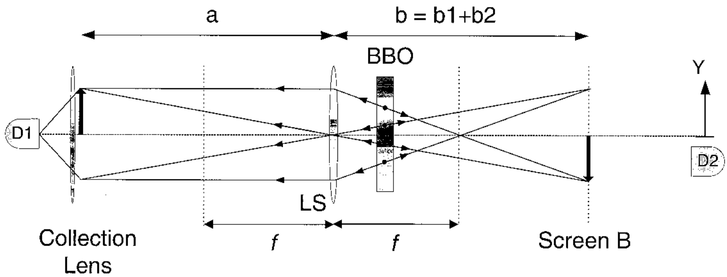

Fig. 3. The unfolded schematic of the experiment. This is equivalent to assume $\mathbf{k}_s + \mathbf{k}_i = 0$ but without losing the important entanglement feature of the momentum conservation of the signal-idler pair. It is clear that the locations of slit A, lens LS, and the "ghost image" must be governed by the Gaussian thin lens equation, bearing in mind the different propagation directions of the signal-idler by the small arrows on the straight-line two-photon paths.

could make a "ghost image" of slit A at "screen" B, see Fig. 3. The physical principle of the two-photon "ghost image" has been reported in Ref. 5.

The experimental condition specified in Popper's experiment is then achieved: when slit A is adjusted to a certain narrow width and slit B is wide open, slit A provides precise knowledge about position of photon 1 on the y-axis up to an accuracy Δy which equals the width of slit A and the corresponding "ghost image" of pinhole A at "screen" B determines the precise position y of photon 2 to within the same accuracy Δy. Δp_y of "photon 2" can be independently studied by measuring the width of its "diffraction pattern" at a certain distance from "screen" B. This is obtained by recording coincidences between detectors D_1 and D_2 while scanning detector D_2 along its y-axis which is behind "screen" B at a certain distance. Instead of a battery of Geiger counters, in our experiment only two photon counting detectors D_1 and D_2 placed behind the respective slits A and B are used for the coincidence detection. Both D_1 and D_2 are driven by step motors and so can be scanned along their y-axes. $\Delta y \, \Delta p_y$ of "photon 2" is then readily calculated and compared with h.[6]

The use of "point source" in the original proposal has been much criticized and considered as the fundamental mistake Popper made.[7, 8] The major objection is that a point source can never produce a pair of entangled particles which preserves two-particle momentum conservation. However, notice that a "point source" is *not* a necessary requirement for Popper's experiment. What is required is the position entanglement of the two-particle system: if the position of particle 1 is precisely known, the position of particle 2 is also 100% determined. So one can learn the precise knowledge of a particle's position through quantum entanglement. Quantum mechanics does allow the position entanglement for an entangled system (EPR state) and there are certain practical mechanisms, such as the "ghost-image" effect shown in our experiment, that can be used for its realization.

The schematic experimental setup is shown in Fig. 4 with detailed indications of the various distances. A CW Argon ion laser line of $\lambda_p =$ 351.1 nm is used to pump a 3 mm long beta barium borate (BBO) crystal for type II SPDC[10] to generate an orthogonally polarized signal-idler photon pair. The laser beam is about 3 mm in diameter with a diffraction limited divergence. It is important not to focus the pump beam so that the phase-matching condition, $\mathbf{k}_s + \mathbf{k}_i = \mathbf{k}_p$, is well reinforced in the SPDC process,[4] where $\mathbf{k}_j$ $(j = s, i, p)$ is the wavevectors of the signal (s), idler (i), and pump (p) respectively. The collinear signal-idler beams, with $\lambda_s = \lambda_i =$ 702.2 nm $= 2\lambda_p$ are separated from the pump beam by a fused quartz dispersion prism, and then split by a polarization beam splitter PBS. The signal beam ("photon 1") passes through the converging lens LS with a 500 mm focal length and a 25 mm diameter. A 0.16 mm slit is placed at location A which is 1000 mm $(=2f)$ behind the lens LS. The use of LS is to achieve a "ghost image" of slit A (0.16 mm) at "screen" B which is at the same optical distance 1000 mm $(=2f)$ from LS, however in the idler beam

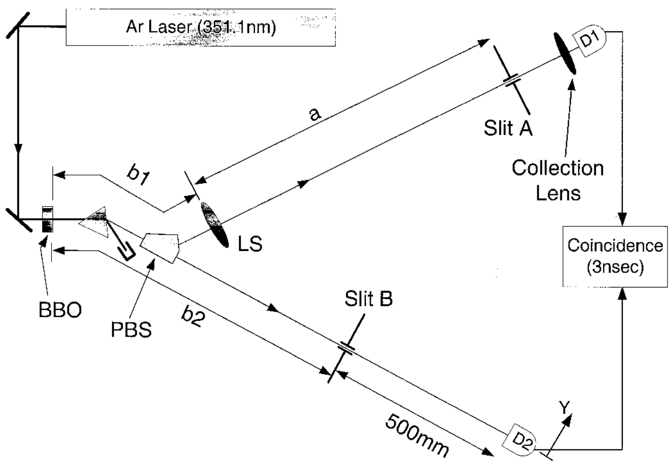

Fig. 4. Schematic of the experimental setup. The laser beam is about 3 mm in diameter. The "phase-matching condition" is well reinforced. Slit A (0.16 mm) is placed 1000 mm $= 2f$ behind the converging lens, LS $(f = 500$ mm). The one-to-one "ghost image" (0.16 mm) of slit A is located at B. The optical distance from LS in the signal beam taken as back through PBS to the SPDC crystal $(b_1 = 255$ mm) and then along the idler beam to "screen B" $(b_2 = 745$ mm) is 1000 mm $= 2f$ $(b = b_1 + b_2)$.

(in the path of "photon 2"). The signal and idler beams are then allowed to pass through the respective slits A and B (a real slit B and then a "ghost image" of slit A) and to trigger the two photon counting detectors D_1 and D_2. A short focal length lens is used with D_1 for collecting the signal beam which passes through slit A. The point-like detector D_2 is located 500 mm behind "screen" B. The detectors are Geiger mode avalanche photodiodes which are 180 μm in diameter. 10 nm band-pass spectral filters centered at 702 nm are used with each of the detectors. The output pulses from the detectors are sent to a coincidence circuit. During the measurements, detector D_1 is fixed behind slit A while detector D_2 is scanned on the y-axis by a step motor.

Measurement 1. We first studied the case in which both slits A and B were adjusted to be 0.16 mm. The y-coordinate of D_1 was chosen to be 0 (center) while D_2 was allowed to scan along its y-axis. The circled dot data points in Fig. 5 show the *coincidence* counting rates against the y-coordinates of D_2. It is a typical single-slit diffraction pattern with $\Delta y \, \Delta p_y = h$. Nothing is special in this measurement except we have learned the width of the diffraction pattern for the 0.16 mm slit and this represents

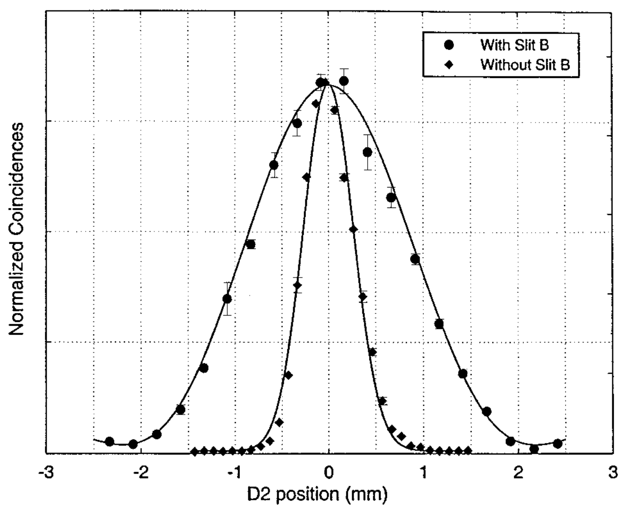

Fig. 5. The observed coincidence patterns. The y-coordinate of D_1 was chosen to be 0 (center) while D_2 was allowed to scan along its y-axis. Circled dot points: *Slit* A = *Slit* B = 0.16 mm. Diamond dot points: *Slit* A = 0.16 mm, *Slit B wide open*. The width of the sinc-square function curve fitted to the circled dot points is a measure of the minimum Δp_y determined by a 0.16 mm slit.

the minimum uncertainty of Δp_y.[6] We should remark at this point that the *single* detector counting rates of D_2 is basically the same as that of the coincidence counts except for a higher counting rate.

Measurement 2. The same experimental conditions were maintained except that slit B was left wide open. This measurement is a test of Popper's prediction. The y-coordinate of D_1 was chosen to be 0 (center) while D_2 was allowed to scan along its y-axis. Because of entanglement of the signal-idler photon pair and the coincidence measurement, only those twins which have passed through slit A and the "ghost image" of slit A at "screen" B with an uncertainty of $\Delta y = 0.16$ mm (which is the same width as the real slit B we have used in measurement 1) would contribute to the coincidence counts through the simultaneous triggering of D_1 and D_2. The diamond dot data points in Fig. 5 report the measured coincidence counting rates against the y coordinates of D_2. The measured width of the pattern is narrower than that of the diffraction pattern shown in measurement 1. At the same time, the width of the pattern is found to be much narrower than the actual size of the diverging SPDC beam at D_2. It is also interesting to notice that the single counting rate of D_2 keeps constant in the entire scanning range, which is very different from that in measurement 1. The experimental data has provided a clear indication of $\Delta y\ \Delta p_y < h$ in the coincidence measurements.

4. QUANTUM MECHANICAL PREDICTION

Given that $\Delta y\ \Delta p_y < h$, is this a violation of uncertainty principle? Before drawing any conclusion, let us first examine what quantum mechanics predicts. If quantum mechanics does provide a solution with $\Delta y\ \Delta p_y < h$ for "photon 2." Indeed, we would be forced to face a paradox as EPR had pointed out in 1935.

We begin with the question: how does one learn the precise position knowledge of photon 2 at "screen" B quantum mechanically? Is it really 0.16 mm as determined by the width of slit A? The answer is in the positive. Quantum mechanics predicts a "ghost" image of slit A at "screen" B which is 0.16 mm for the above experimental setup. The crucial point is we are dealing with an entangled two-photon state of SPDC,[3, 9]

$$|\Psi\rangle = \sum_{s,i} \delta(\omega_s + \omega_i - \omega_p)\, \delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p)\, a_s^\dagger(\omega(\mathbf{k}_s))\, a_i^\dagger(\omega(\mathbf{k}_i))\, |0\rangle \tag{1}$$

where ω_j, $\mathbf{k}_j$ ($j = s, i, p$) are the frequencies and wavevectors of the signal (s), idler (i), and pump (p) respectively. ω_p and $\mathbf{k}_p$ can be considered as constants while $a_s^\dagger$ and $a_i^\dagger$ are the respective creation operators for the signal and the idler. As given in the above form, the entanglement feature in state (1) may be thought of as the superposition of infinite numbers of "two-photon" states that corresponds to the infinite numbers of ways the SPDC signal-idler can satisfy the conditions of energy and momentum conservation, as represented by the δ-functions of the state which is technically known as phase-matching conditions:

$$\omega_s + \omega_i = \omega_p, \qquad \mathbf{k}_s + \mathbf{k}_i = \mathbf{k}_p \tag{2}$$

It is interesting to see that even though there is no precise knowledge of the momentum for either the signal or the idler, the state nonetheless provides precise knowledge of the *momentum correlation* of the pair. In the language of EPR, the momentum for neither the signal photon nor the idler photon is determined but if a measurement on one of the photons yields a certain value, the momentum of the other photon is 100% determined.

To simplify the physical picture, we "unfold" the signal-idler paths in the schematic of Fig. 4 into that shown in Fig. 3, which is equivalent to assuming $\mathbf{k}_s + \mathbf{k}_i = 0$ while not losing the important entanglement feature of the momentum conservation of the signal-idler pair. This important peculiarity selects the only possible optical paths of the signal-idler pairs that result in a "click-click" coincidence detection which are represented by *straight lines* in this unfolded version of the experimental schematic so that the "image" of slit A is well-produced in *coincidences* as shown in the figure. It is similar to an optical imaging in the "usual" geometric optics picture, bearing in mind the different propagation directions of the signal-idler indicated by the small arrows on the *straight lines*. It is easy to see that a ""clear" image requires the locations of slit A, lens LS, and screen B to be governed by the Gaussian thin lens equation,[5]

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \tag{3}$$

In our experiment, we have chosen $a = b = 2f = 1000$ mm, so that the "ghost image" of slit A at "screen" B must have the same width as that of slit A. The measured size of the "ghost image" agrees with theory.

In Fig. 3 we see clearly these two-photon paths (*straight lines*) that result in a "click-click" joint detection are restricted by slit A, lens LS as well as momentum conservation. As a result, any signal-idler pair that passes through the 0.16 mm slit A would be "localized" within $\Delta y =$ 0.16 mm at "screen" B. In this way, one does learn the precise position knowledge of photon 2 through the entanglement nature of the two-photon system.

One could also explain this “ghost image” in terms of conditional measurements: conditioned on the detection of “photon 1” by detector D_1 behind slit A, “photon 2” can only be found in a certain position. In other words, “photon 2” is localized only upon the detection of photon 1.

Now let us go further to examine Δp_y of photon 2 which is conditionally “localized'' within $\Delta y = 0.16$ mm at “screen” B. In order to study Δp_y, the photon counting detector D_2 is scanned 500 mm behind “screen” B to measure the “diffraction pattern”. Δp_y can be easily estimated from the measurement of the width of the diffraction pattern.[6] The two-photon paths, indicated by the *straight lines*, reach detector 2 which is located 500 mm behind “screen” B so that detector D_2 will receive “photon 2” in a much narrower width under the condition of the “click” of detector D_1 as shown in measurement 2, unless a real physical slit B is applied to “disturb” the *straight lines*.

Apparently we have a paradox: quantum mechanics provides us with a solution which gives $\Delta y \; \Delta p_y < h$ in measurement 2 and the experimental measurements agree with the prediction of quantum mechanics.

5. CONCLUSION

It is the same paradox of EPR. Indeed, one could consider this experiment as a variant of the 1935 EPR *gedankenexperiment* in which the position-momentum uncertainty was questioned by Einstein–Podolsky–Rosen based on the discussion of a two-particle entangled state.[2] Comparing with the EPR-Bohm experiment,[11] which is a simplified version of the 1935 EPR *gedankenexperiment*, the spin for neither particle is determined (uncertain); however, if one particle is measured to be spin up along a certain direction, the other one must be spin down along that direction (certain). All the spin components of a particle can be precisely determined through the measurement of its twin.

Quantum mechanics gives prediction for the EPR and the EPR-Bohm correlations based on the measurements for entangled states. All reported historical experiments have shown good agreement with quantum mechanics as well as EPR’s prediction (but not their interpretation). The results of our experiment agree with quantum mechanics and Popper’s prediction too. We therefore consider the following discussions may apply to both EPR and Popper.

Popper and EPR were correct in the prediction of the physical outcomes of their experiments. However, Popper and EPR made the same error by applying the results of two-particle physics to the explanation of the behavior of an individual particle. The two-particle entangled state is not the state of two individual particles. Our experimental result is emphatically NOT a violation of the uncertainty principle which governs the behavior of an individual quantum.

In both the Popper and EPR experiments the measurements are "joint detection" between two detectors applied to entangled states. Quantum mechanically, an entangled two-particle state only provides *the precise knowledge of the correlations of the pair*. Neither of the subsystems is determined by the state. It can be clearly seen from our above analysis of Popper's experiment that this kind of measurements is only useful to decide on how good the correlation is between the entangled pair. In other words, the behavior of "photon 2" observed in our experiment is conditioned upon the measurement of its twin. A quantum must obey the uncertainty principle but the "conditional behavior" of a quantum in an entangled two-particle system is different. The uncertainty principle is not for "conditional" behavior. We believe paradoxes are unavoidable if one insists the *conditional behavior* of a particle is the *behavior* of a particle. This is the central problem of the rationale behind both Popper and EPR. $\Delta y \, \Delta p_y \geqslant h$ is not applicable to the conditional behavior of either "photon 1" or "photon 2" in the case of the Popper and EPR type of measurements.

The behavior of photon 2 conditioned upon photon 1 is well represented by the two-photon amplitudes. Each of the *straight lines* in the above discussion corresponds to a two-photon amplitude. Quantum mechanically, the superposition of these two-photon amplitudes are responsible for a "click-click" measurement of the entangled pair. A "click-click" joint measurement of the two-particle entangled state projects out certain two-particle amplitudes and only these two-particle amplitudes feature in the quantum formalism. In the above analysis we never consider "photon 1" or "photon 2" *individually*. Popper's question about the momentum uncertainty of photon 2 is then inappropriate. The correct question to ask in these measurements should be: what is the Δp_y for the signal-idler pair which are "localized" within $\Delta y = 0.16$ mm at "screen" B and at "screen" A and governed by the momentum conservation? This is indeed the central point for this experiment. There is no reason to expect the "conditionally localized photon 2" will follow the familiar interpretation of the uncertainty relation as shown in Fig. 5.

Quantum mechanics shows that the superposition of these two-photon amplitudes results in a non-factorizable two-dimensional *biphoton* wavepacket[9, 12, 13] instead of two individual wavepackets associated with photon 1 and photon 2. Figure 6 gives a simple picture of the *biphoton* wavepacket of SPDC. We believe all the problems raised by the EPR and Popper type experiments can be duly resolved if the concept of *biphoton* is adopted in place of two individual photons.

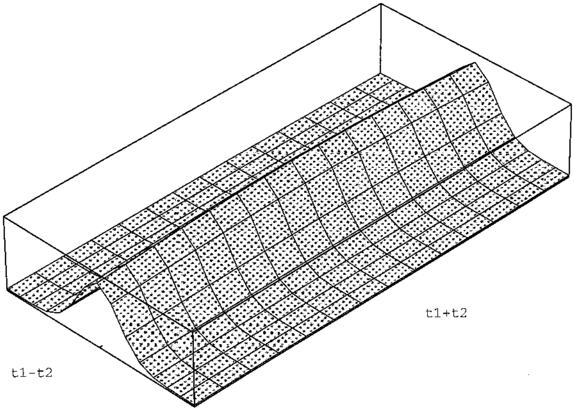

Fig. 6. Biphoton wavepacket envelope calculated from the state of type I spontaneous parametric down conversion. For a simplified situation, it can be written as: $\Psi(t_1, t_2) = A_0 e^{-\sigma_+^2 (t_1 + t_2)^2} e^{-\sigma_-^1 (t_1 - t_2)^2} e^{-i\Omega_s t_1} e^{-i\Omega_i t_2}$, where Ω_j, $j = s, i$, is the central frequency for signal or idler, $1/\sigma_\pm$ are coherence times, $t_i \equiv T_i - L_i/c$, $i = 1, 2$, T_i is the detection time of detector i and L_i the optical pathlength of the signal or idler from SPDC to the ith detector. $\Psi(t_1, t_2)$ is a non-factorizable two-dimensional wavepacket, we may call it *biphoton*.

Once again, this recent demonstration of the thought experiment of Popper calls our attention to the important message: the physics of the entangled two-particle system must inherently be very different from that of individual particles. In the spirit of the above discussions, we conclude that it has been a long-standing historical mistake to mix up the uncertainty relations governing an individual single particle with an entangled two-particle system.

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London, 1983). Amongst the most notable opponents to the "Copenhagen School" were Einstein–Podolsky–Rosen, de Broglie, Landé, and Karl Popper. One may not agree with Popper's philosophy (EPR classical reality as well) but once again, Popper's thought experiment brings yet attention to the fundamental problems of quantum theory.

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